

Session 27

Ground↔Grid - Simple, Right?

2025 WSLs Annual Institute

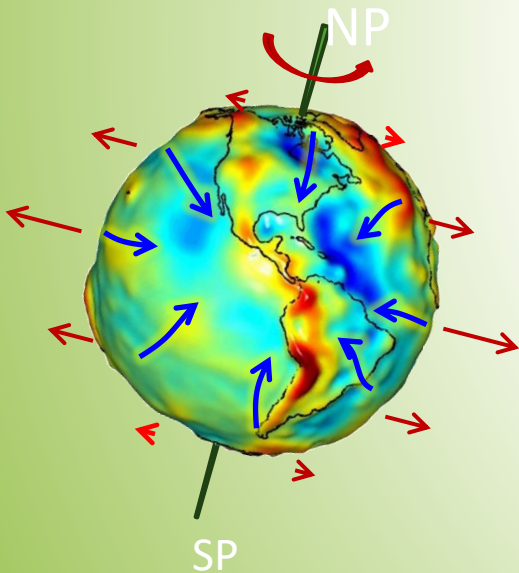
23 January 2005

Jerry Mahun, PLS

Thrice-retired

jerry.mahun@gmail.com

<https://jerrymahun.com>



I. Spatial Systems

II. Distortions

III. Earth Models

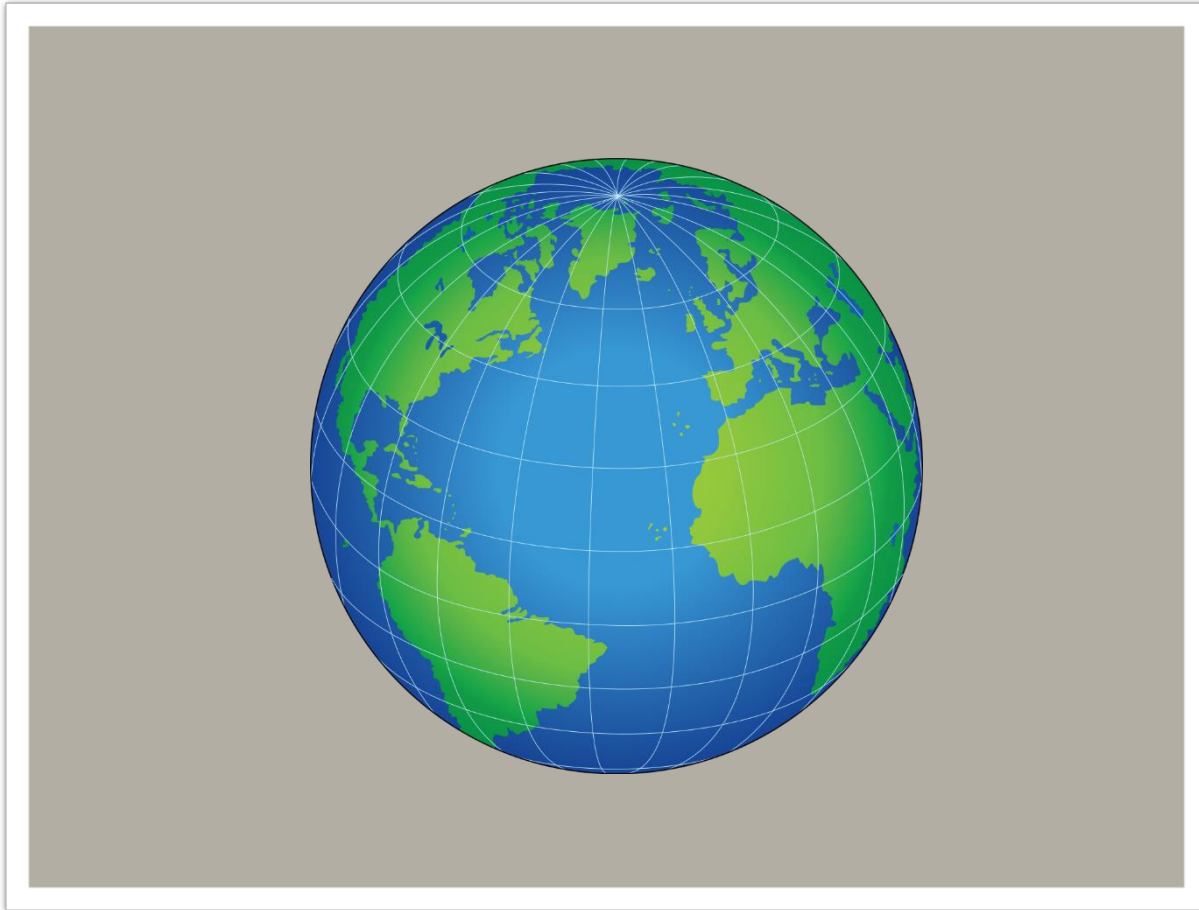
IV. Creating a Grid

V. Wisconsin Coordinate Systems

VI. Ground and Grid

VII. Grids and PLSS Lost Corners

Grid ↔ Ground - Simple, Right?



I. Spatial Systems

A. Three-Dimensional

1. Geodetic Coordinates

Reference defined by ellipsoid and fit.

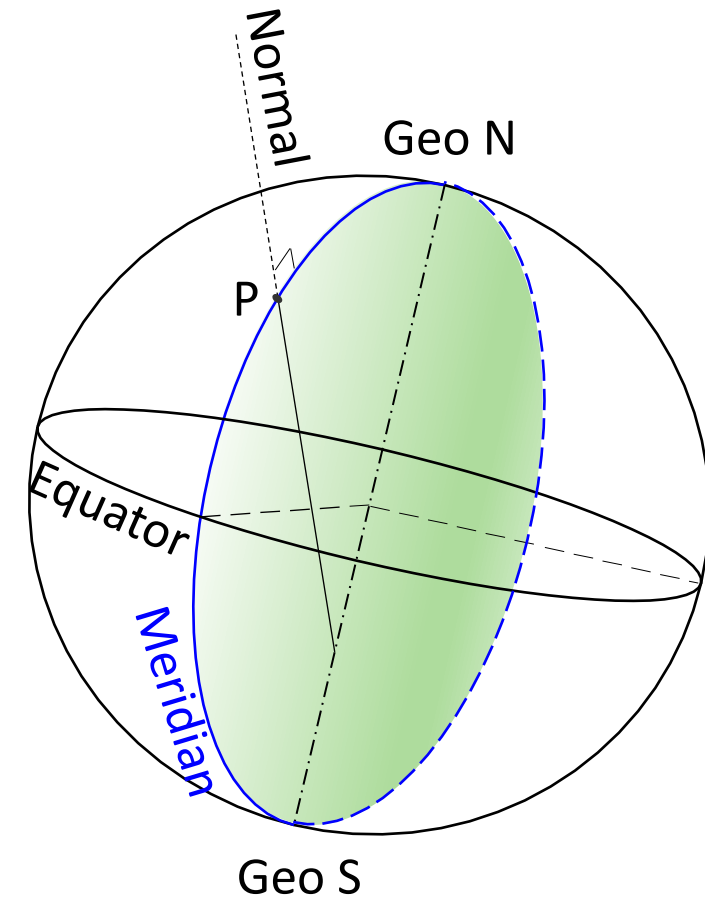
NAD 83 - GRS 80 fit to Earth's mass center.

NAD 27 - Clarke 1866 fit to Meades Ranch, KS

Normal A line from the observer's position, P, perpendicular to the ellipsoid

Meridian An elliptical section containing the normal and semi-minor axes. Defines Geodetic N at a point.

Geodetic meridians converge.

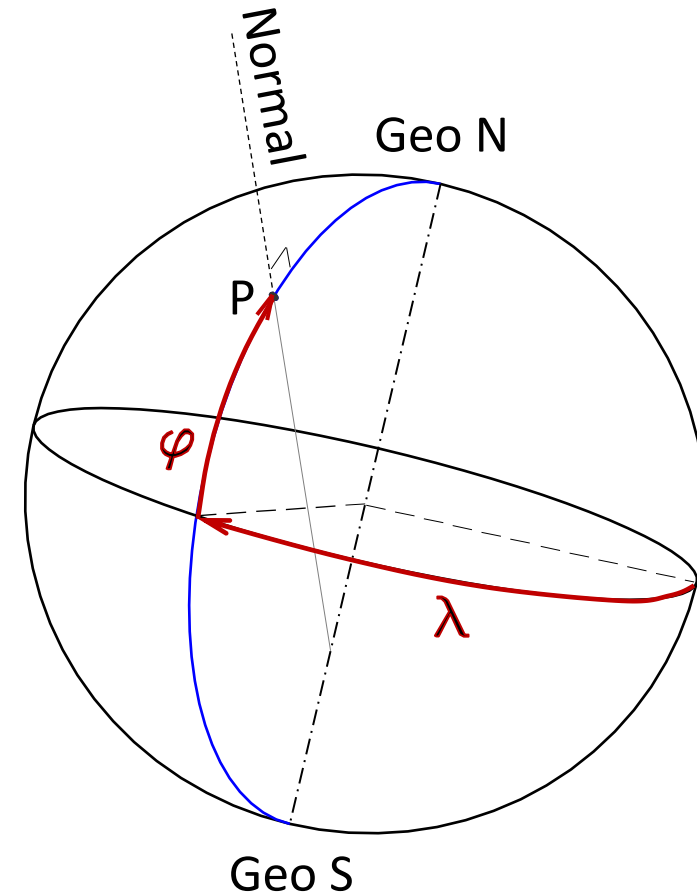


A. Three-Dimensional

1. Geodetic Coordinates

Longitude (λ) - Angle in Equatorial plane
E or W from Greenwich Meridian
 0° - 180° W; 0° - 180° E

Latitude (Φ) - Angle in meridian N or
S of the Equator.
 0° - 90° N; 0° - 90° S



A. Three-Dimensional

1. Geodetic Coordinates

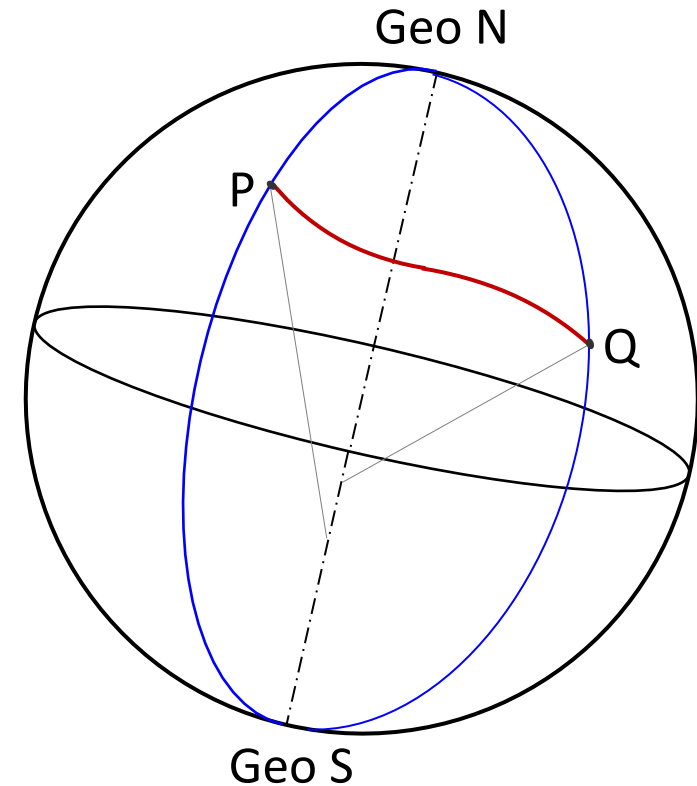
Disadvantages:

Positions are expressed in angular values

Distances are in angular values

Elliptical geometry

Shortest distance between two points
is a *geodesic* - shallow s-shape
curve.



I. Spatial Systems



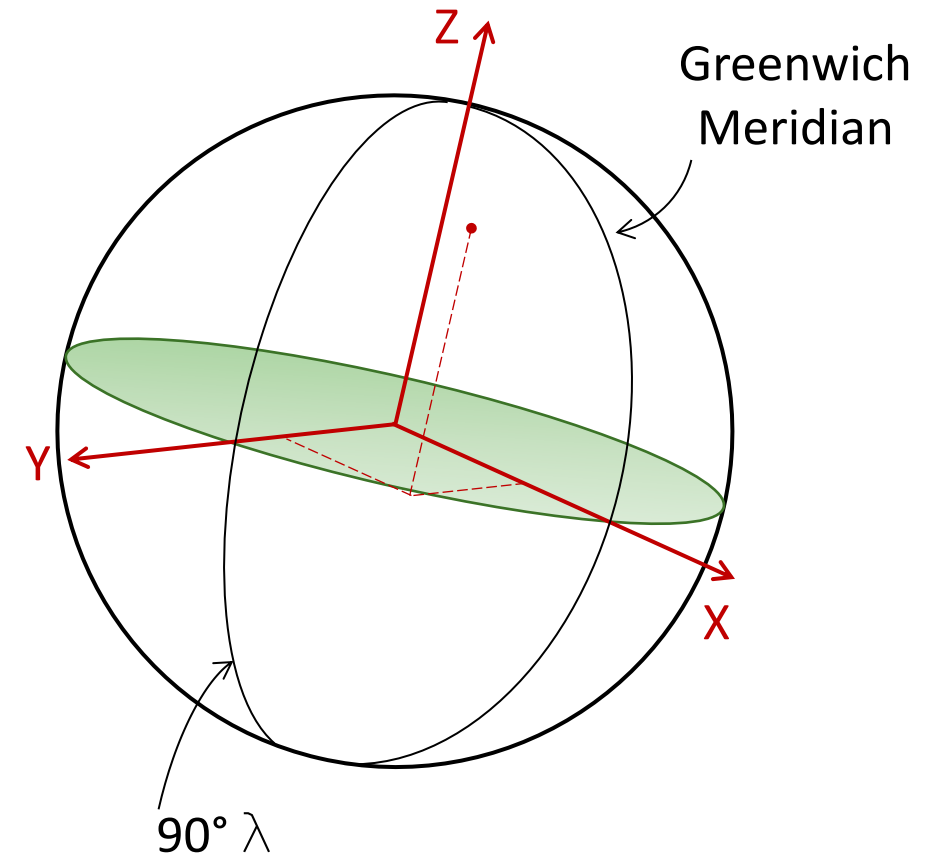
A. Three-Dimensional

2. Terrestrial Coordinate System - TCS

Three axis rectangular system

Origin at Earth's mass center

Coordinates are linear values



A. Three-Dimensional

2. Terrestrial Coordinate System - TCS

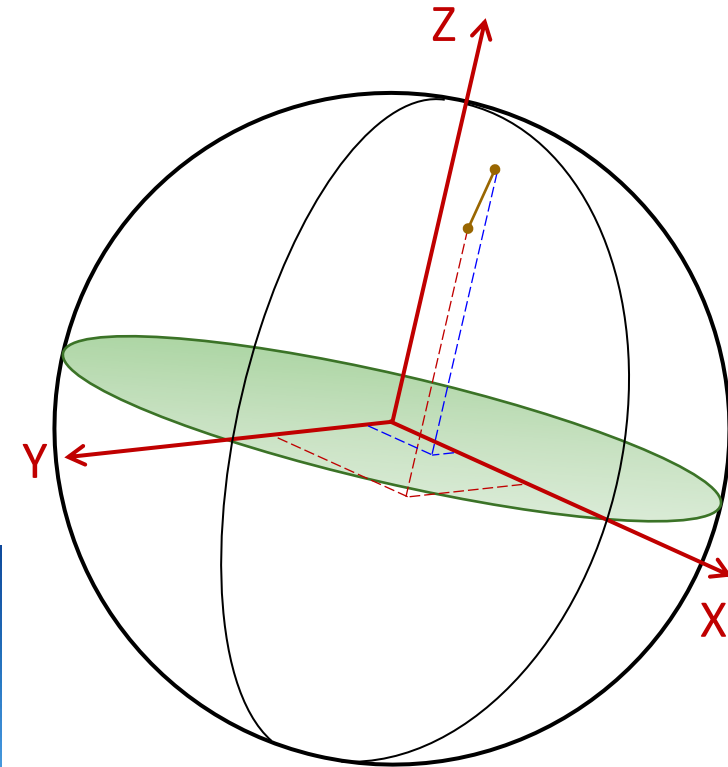
Disadvantages:

Huge coordinate values.

Negative coordinates

No “up” (vertical direction)

Top and bottom of vertical structures
have different 3D coordinates.



I. Spatial Systems



B. Two Dimensional

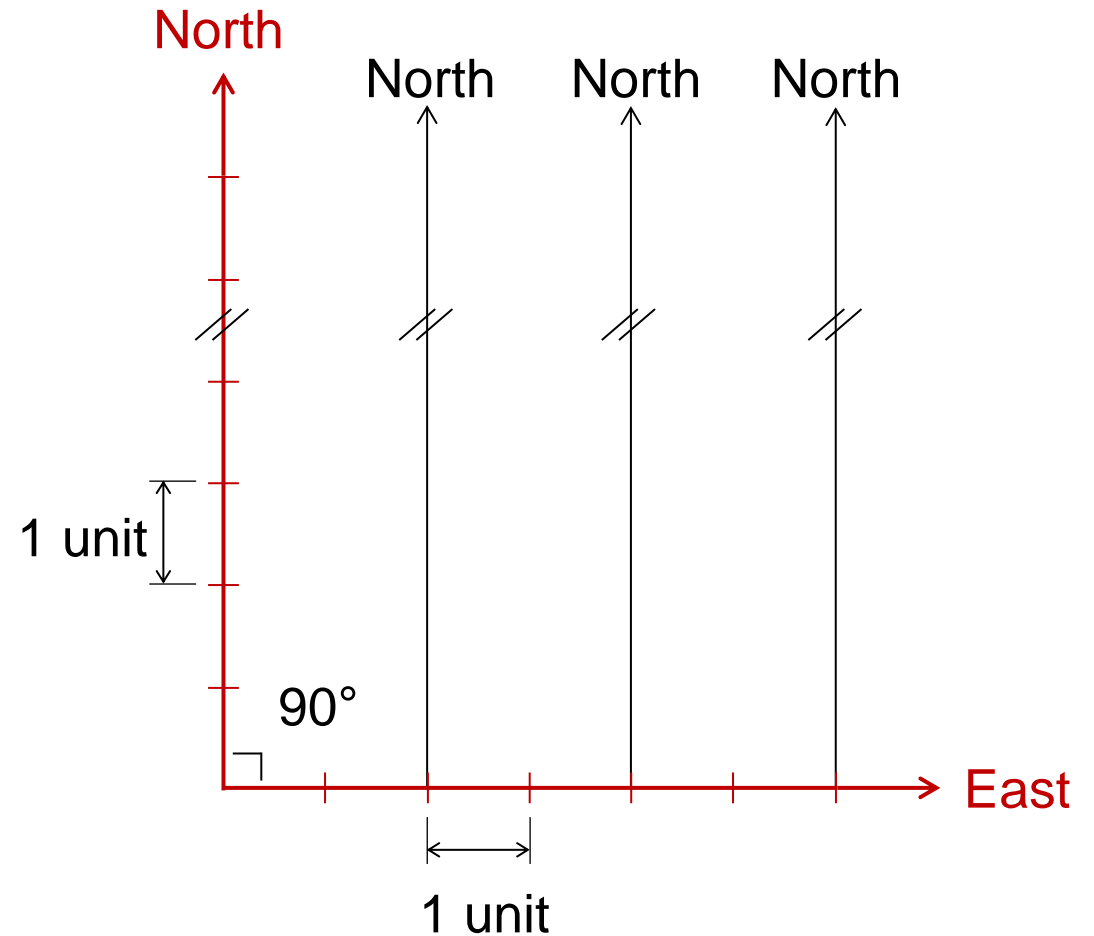
1. Planar

Characteristics

- a. Orthogonal
- b. Parallel north lines
- c. Uniform scale in both directions

Comps are simple.

Disadvantage(s)?



I. Spatial Systems



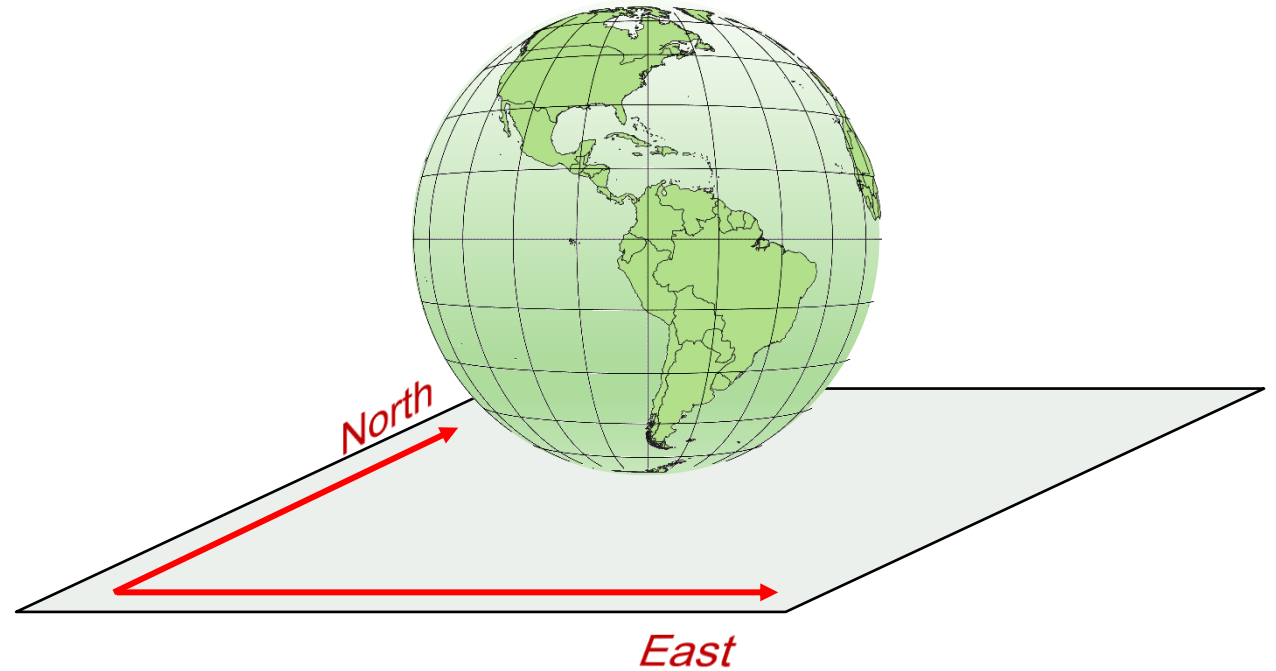
B. Two Dimensional

1. Planar

Disadvantages

We're on a 3D earth

We want to put it in a 2D system



I. Spatial Systems



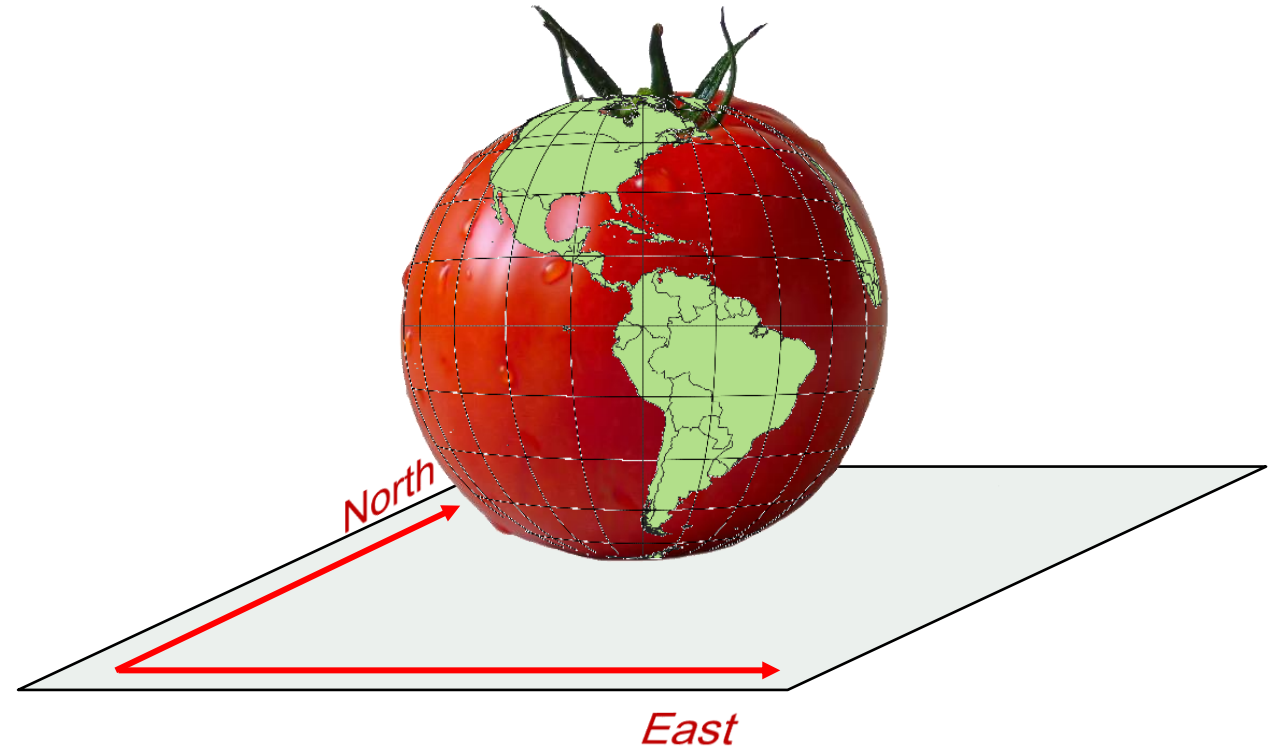
B. Two Dimensional

1. Planar

Disadvantages

We're on a 3D earth

We want to put it in a 2D system



I. Spatial Systems



B. Two Dimensional

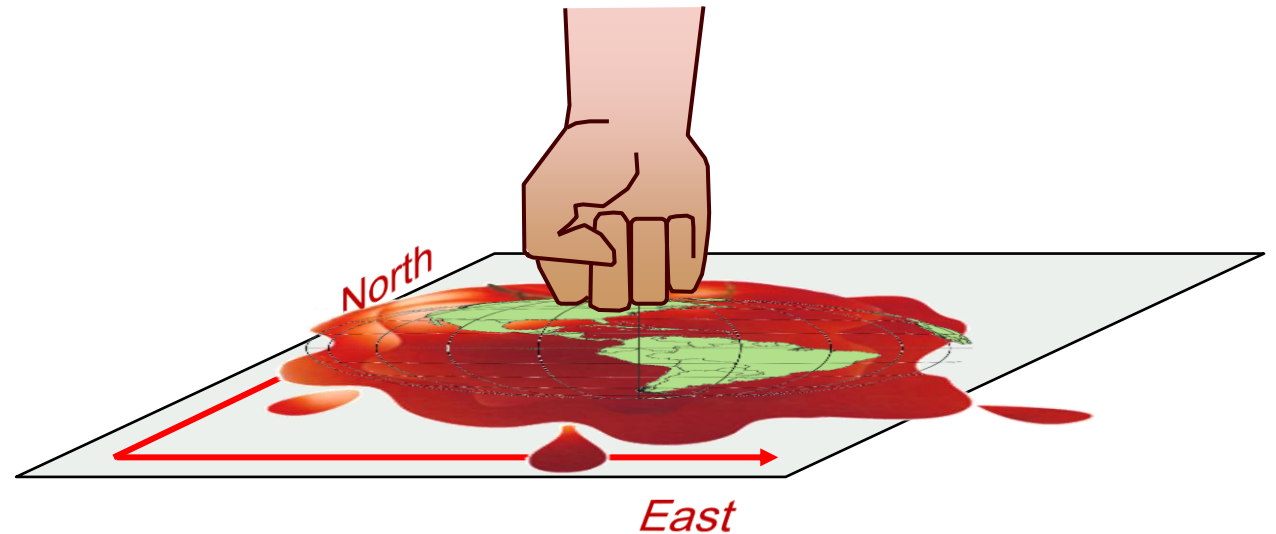
1. Planar

Disadvantages

We're on a 3D earth

We want to put it in a 2D system

With a direct projection we get a distorted representation



I. Spatial Systems



B. Two Dimensional

1. Planar

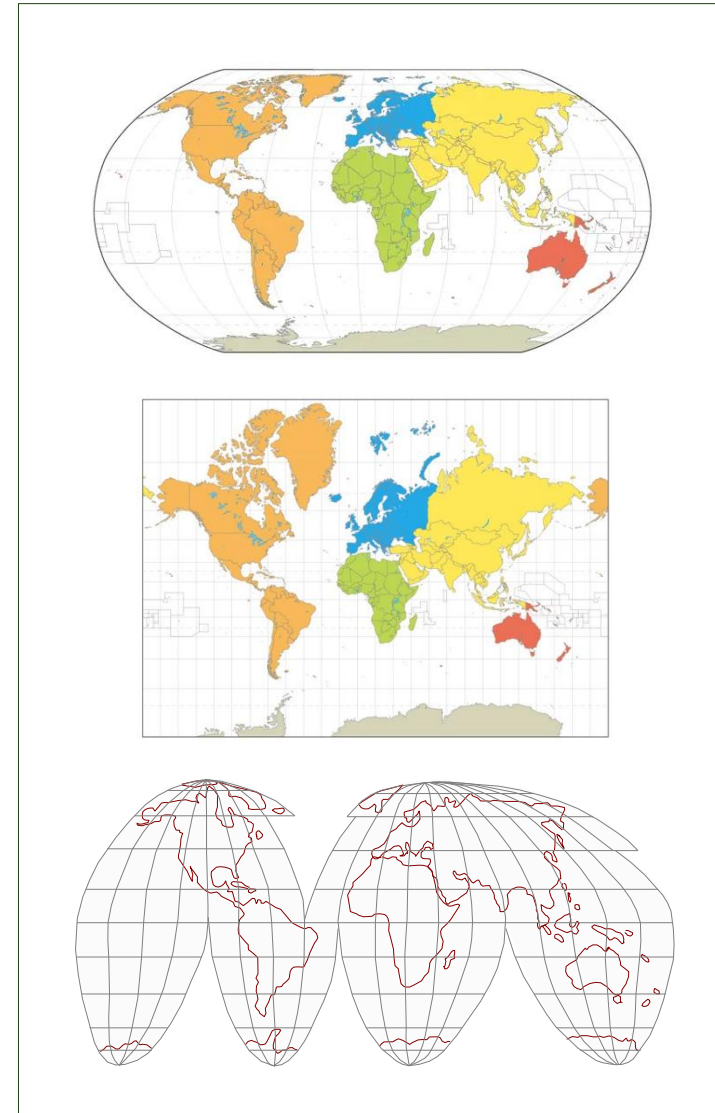
Disadvantages

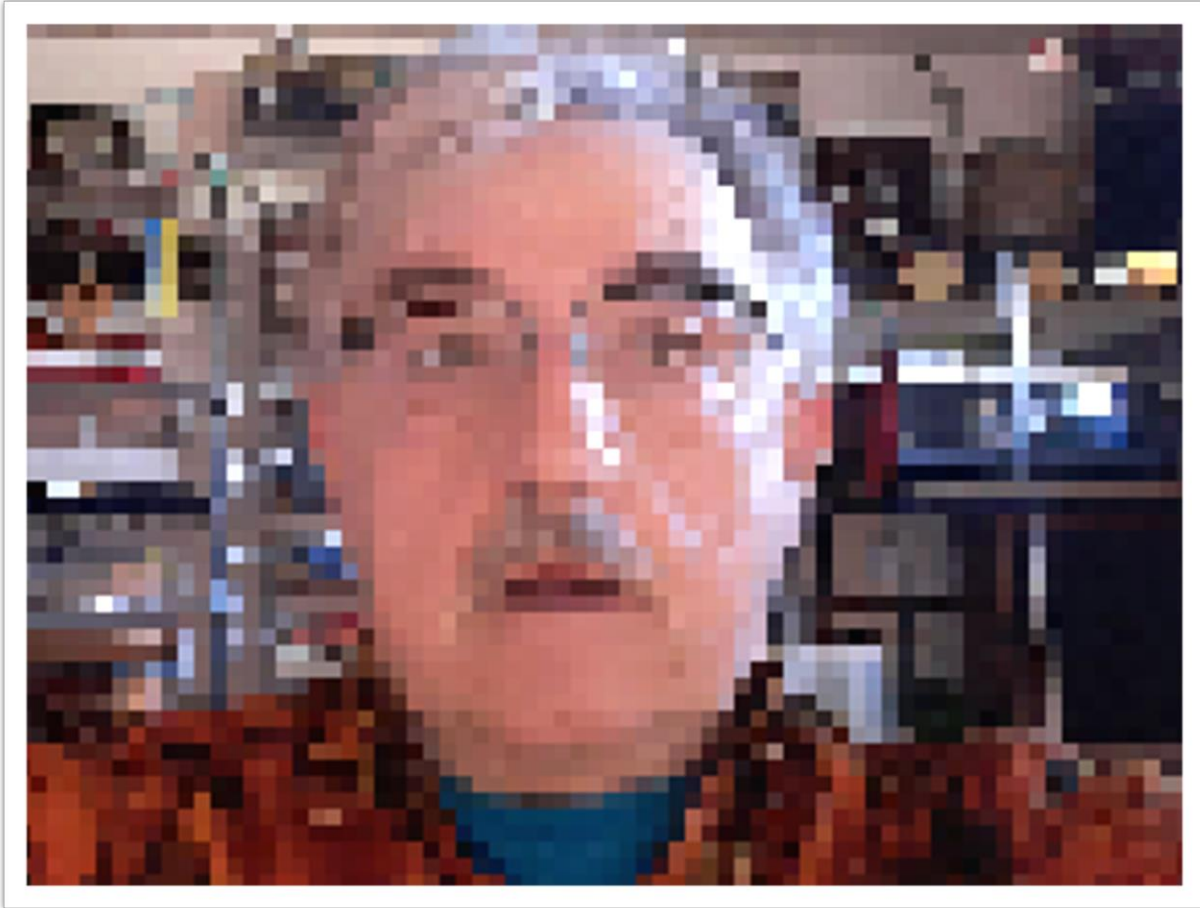
We're on a 3D earth

We want to put it in a 2D system

With a direct projection we get a distorted representation

Different mathematical projections distort different ways.



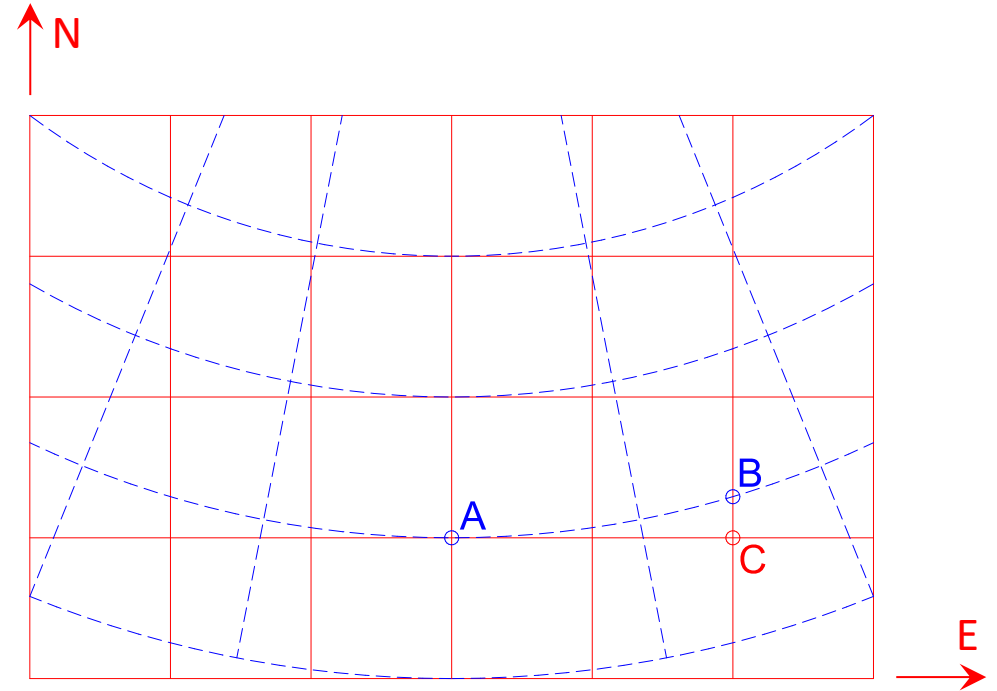
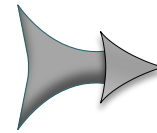


II. Distortions

II. Distortions

A. Projection Area

The smaller the area projected, the smaller the distortions.



Grid (2D) - Solid red
Geodetic (3D) - Dashed blue

II. Distortions

B. Types

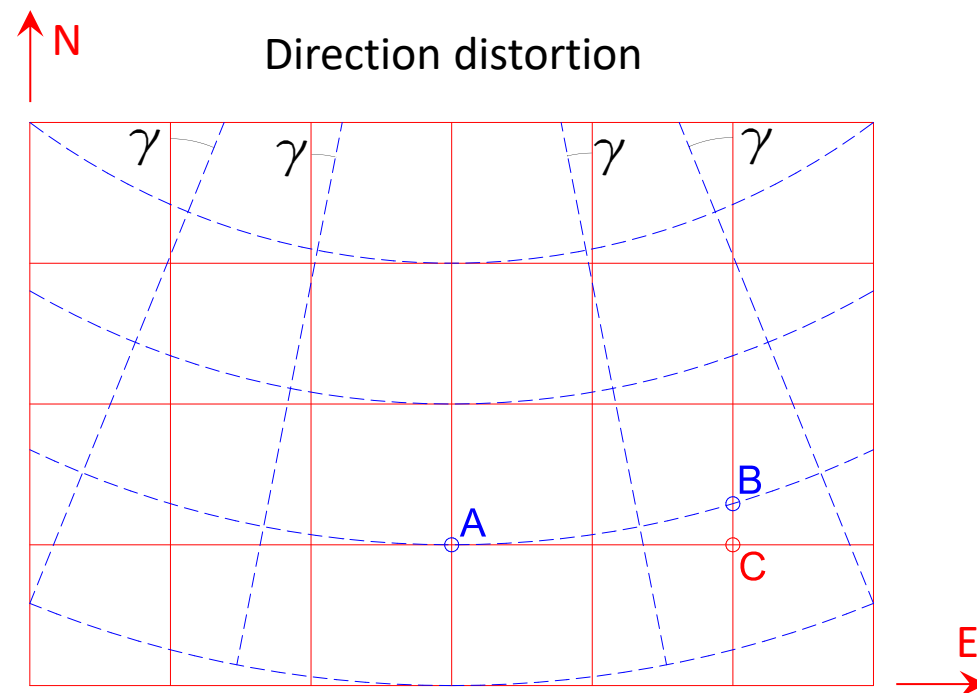
The two primary distortions are

1. Direction

3D meridians converge, 2D do not.
3D E/W lines are curved, 2D are straight.

No distortion at center of projection
Increases moving E & W of center

γ : *convergence*; angle between grid and geodetic north



II. Distortions



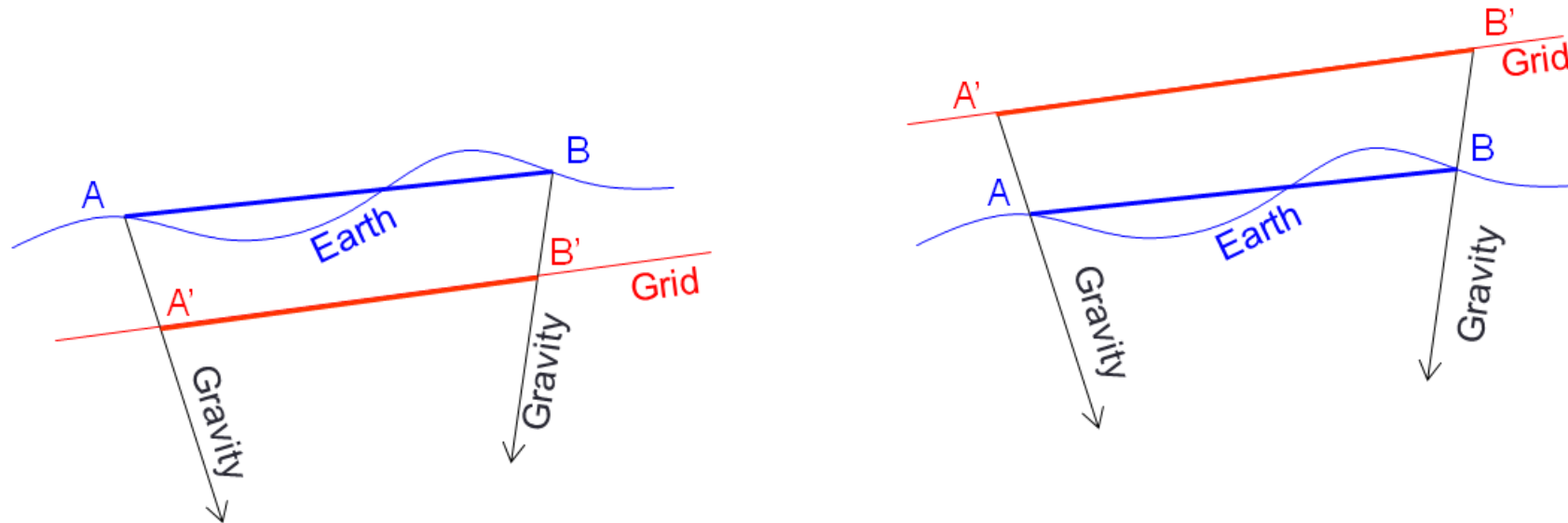
B. Types

The two primary distortions are

2. Distance

Ground points must be projected vertically to the 2D grid plane.

This moves them closer together or further apart, altering distance.



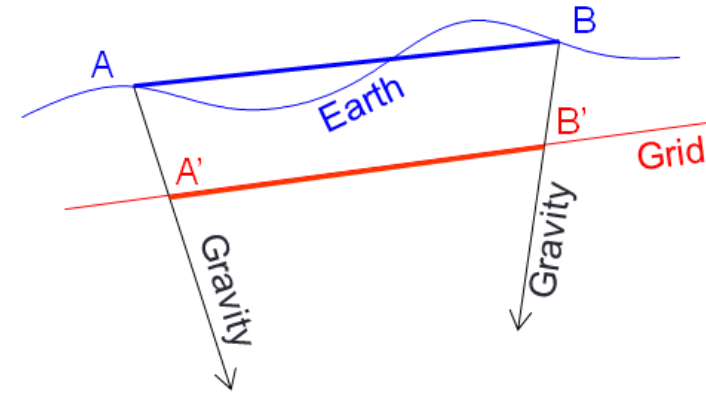
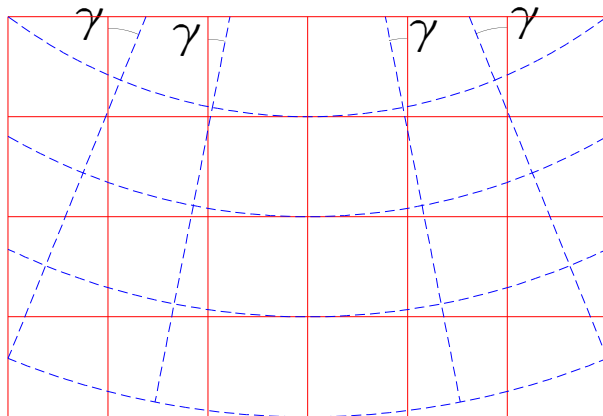
II. Distortions

C. Compensation

Except for *extremely* small areas, projecting 3D to 2D will always create direction and distance distortions.

They are *systematic* errors - can be compensated mathematically *as long as* we know how the surfaces are mathematically connected.

For that, we need some earth models.





III. Earth Models

A. Physical Earth - Ground

The surface on which we measure.

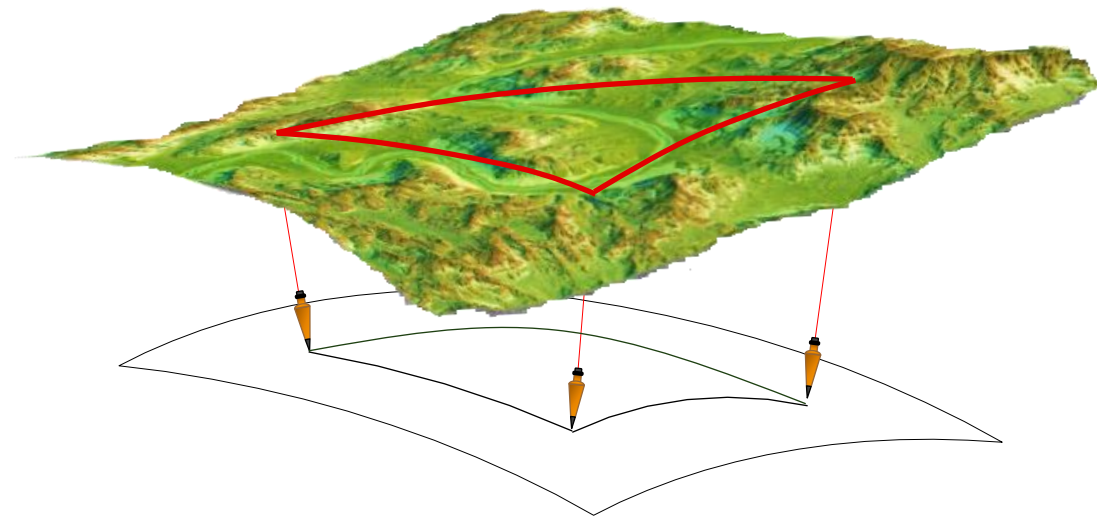
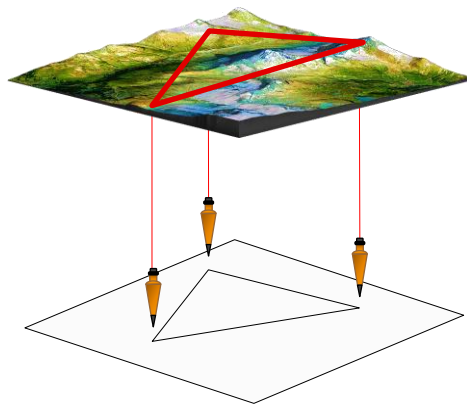
Not mathematical.

Over small areas, we can assume a flat reference system - plane; simple grid

Over larger areas, must account for earth's shape and dynamics - curved reference

Must then project from curved surface to a flat grid.

Need some to connect mathematically



B. Geoid

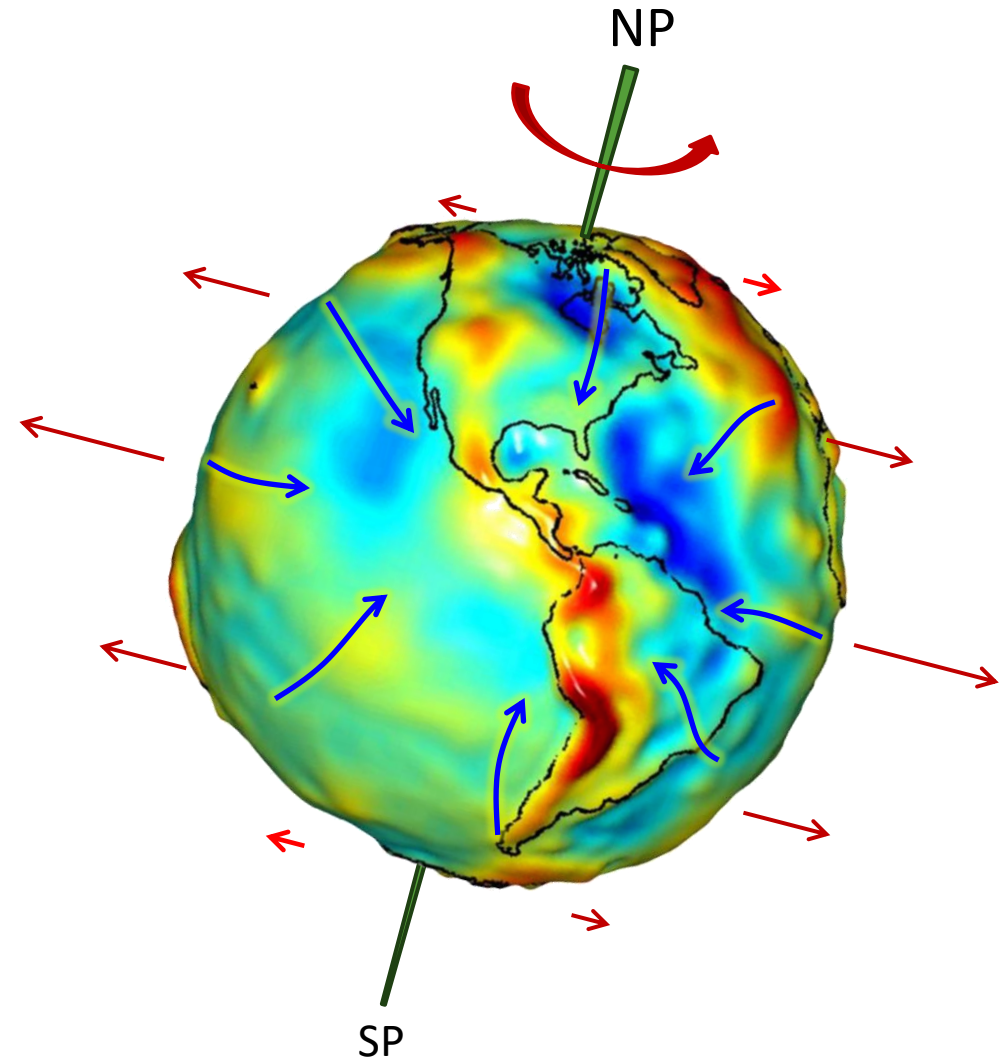
An equipotential surface defined by gravity (in) and centrifugal force (out).

Gravity = $f(\text{mass})$

Earth: non-homogeneous; mass anomalies

⇒ Lines of gravity are neither parallel nor straight.

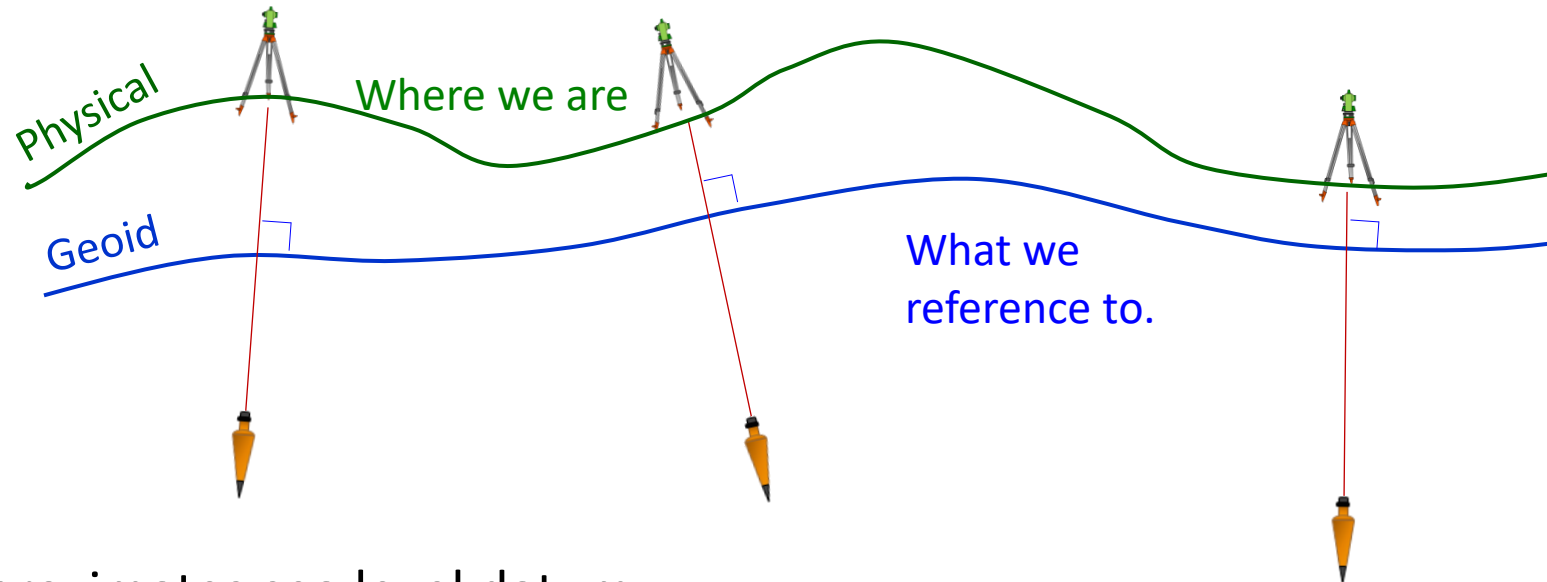
Geoid is a lumpy and changing nonmathematical surface.



B. Geoid

Gravity is perpendicular to the geoid

Centering a bubble or using a plumb bob orients equipment to the geoid.



Geoid approximates sea level datum

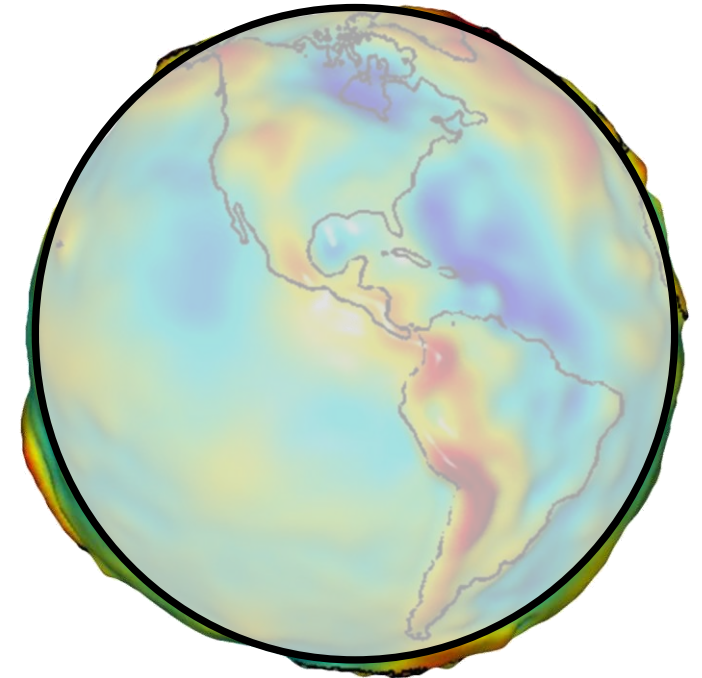
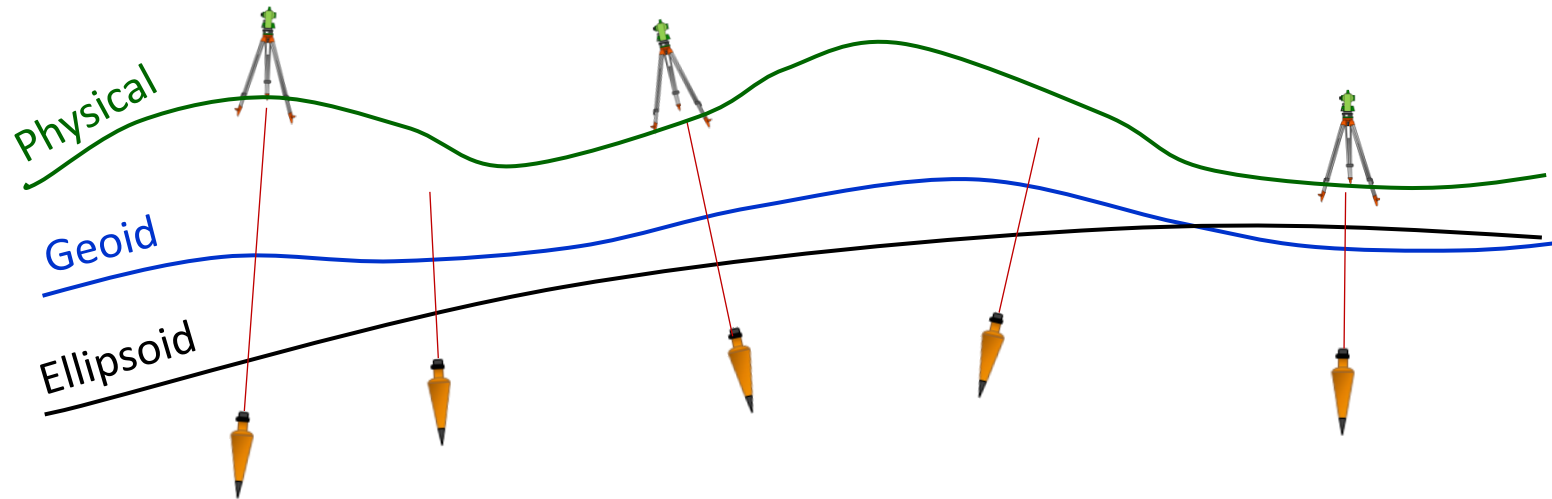
Connected to physical Earth by elevations - orthometric heights

C. Ellipsoid

Mathematical 3D surface

Fit to geoid

Doesn't fit exactly; compromises

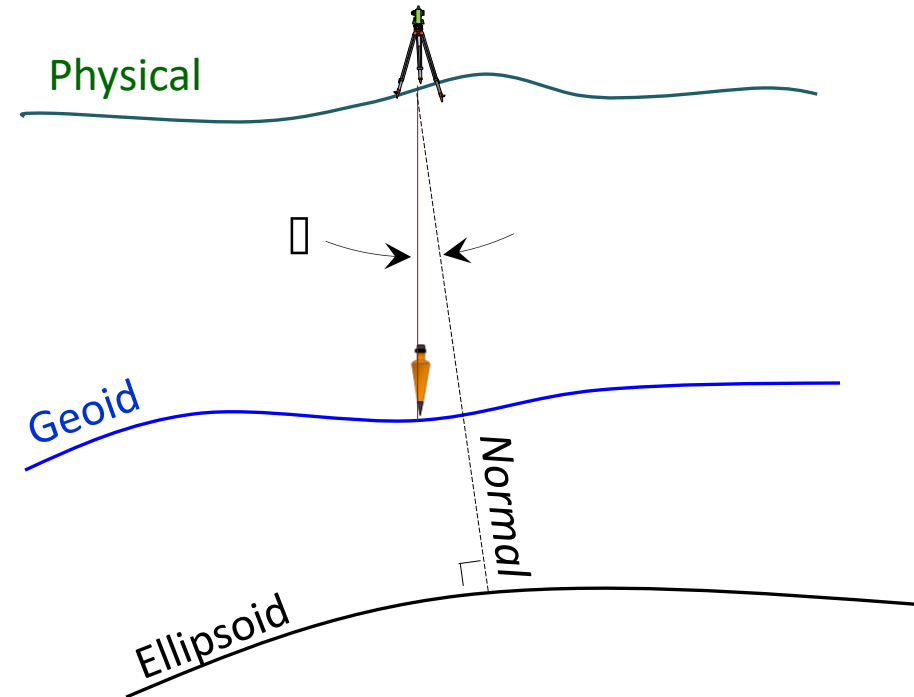


C. Ellipsoid

Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

Skewness - Deflection of the vertical, δ

Angle between directions of gravity and ellipsoid normal.



C. Ellipsoid

Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

Vertical separation

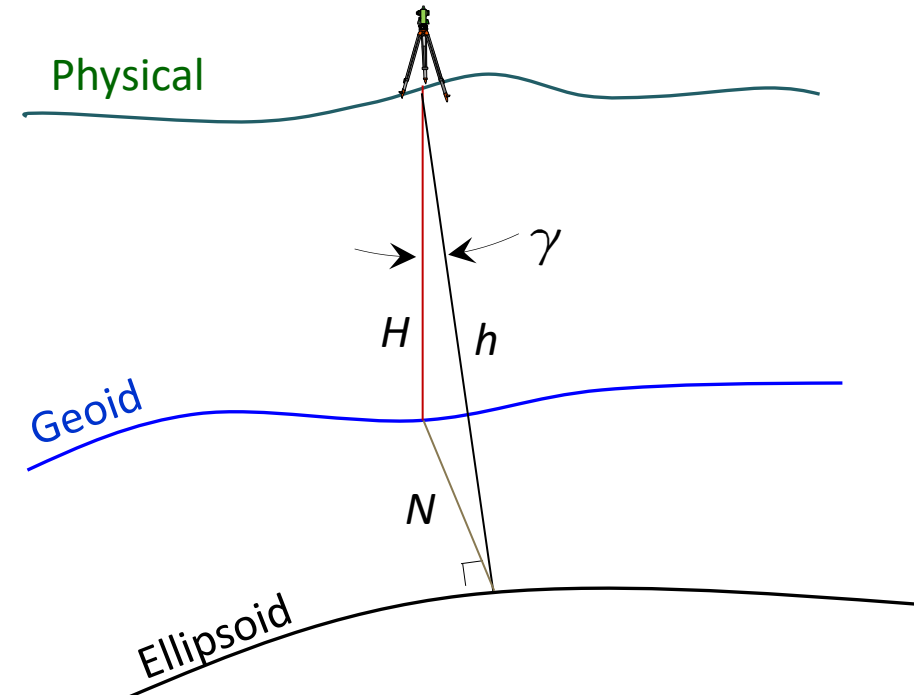
Heights between the surfaces

H - Orthometric: geoid to ground

N - Geoid: ellipsoid to geoid

h - Ellipsoidal: ellipsoid to ground

$$h = H + N$$

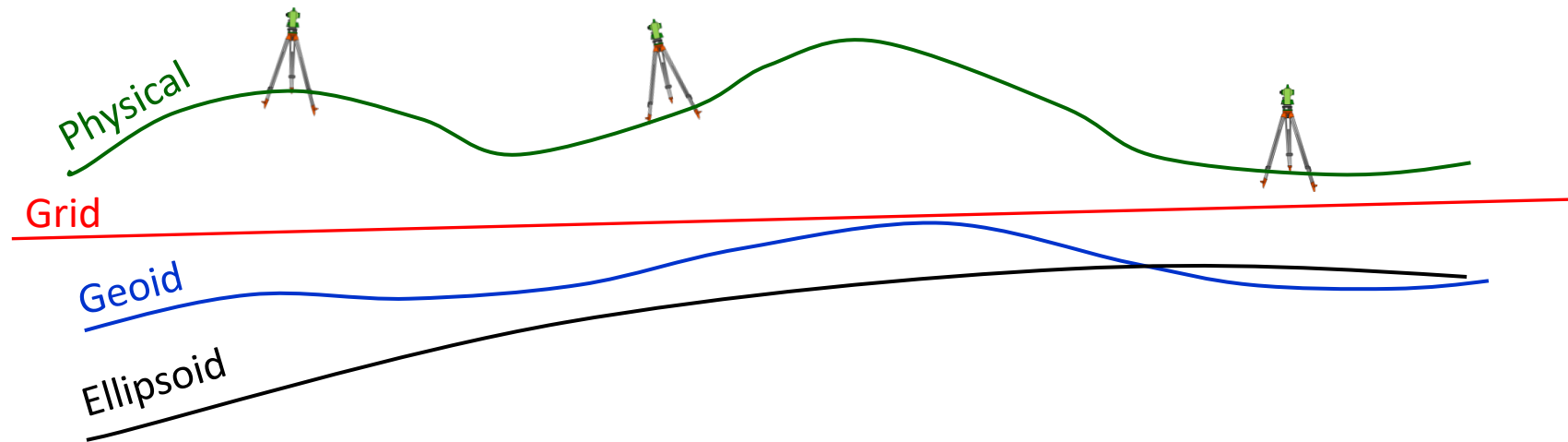


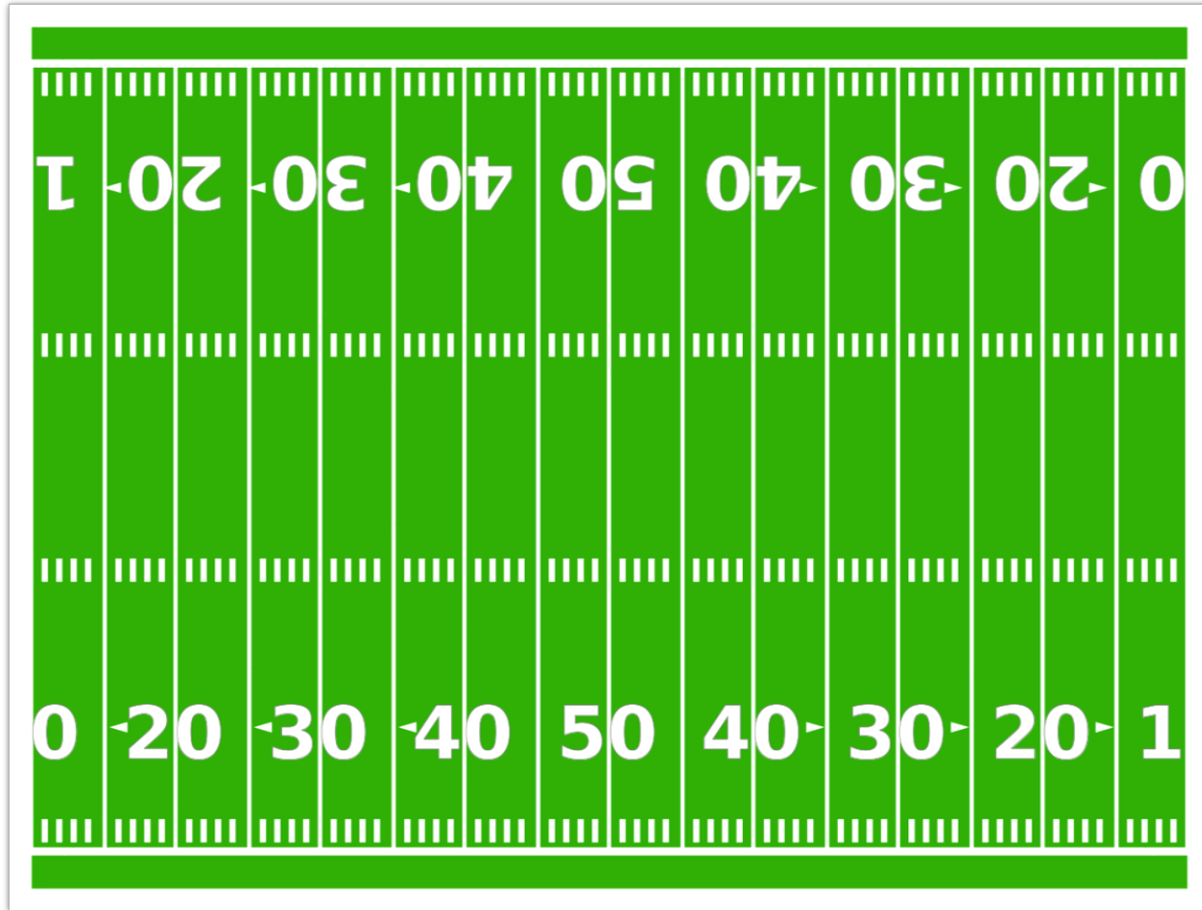
D. Grid

The final surface is a grid

Can be more than one depending on coordinate systems needed.

Move points from ground, through geoid and ellipsoid, to the grid





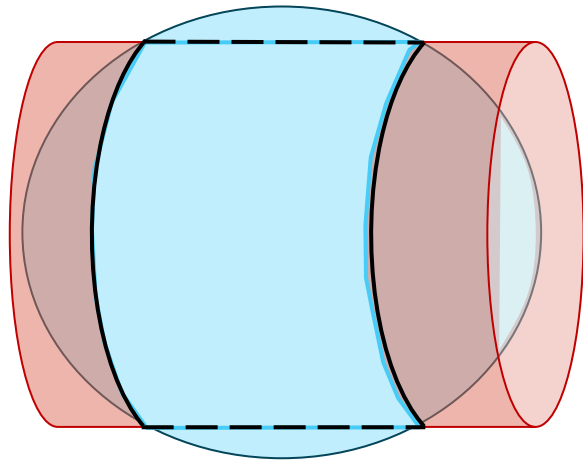
IV. Creating a Grid

IV. Creating a Grid

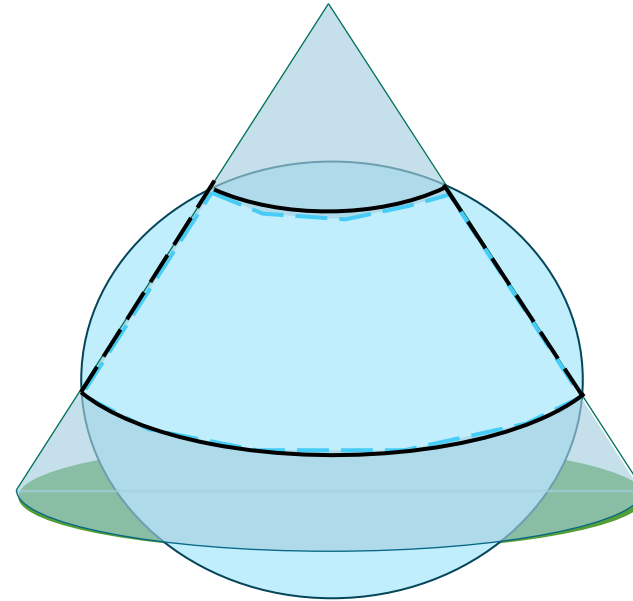


A. Projection Surfaces

Grid is based on a Projection Surface which is fit to Ellipsoid



Cylinder



Cone

IV. Creating a Grid

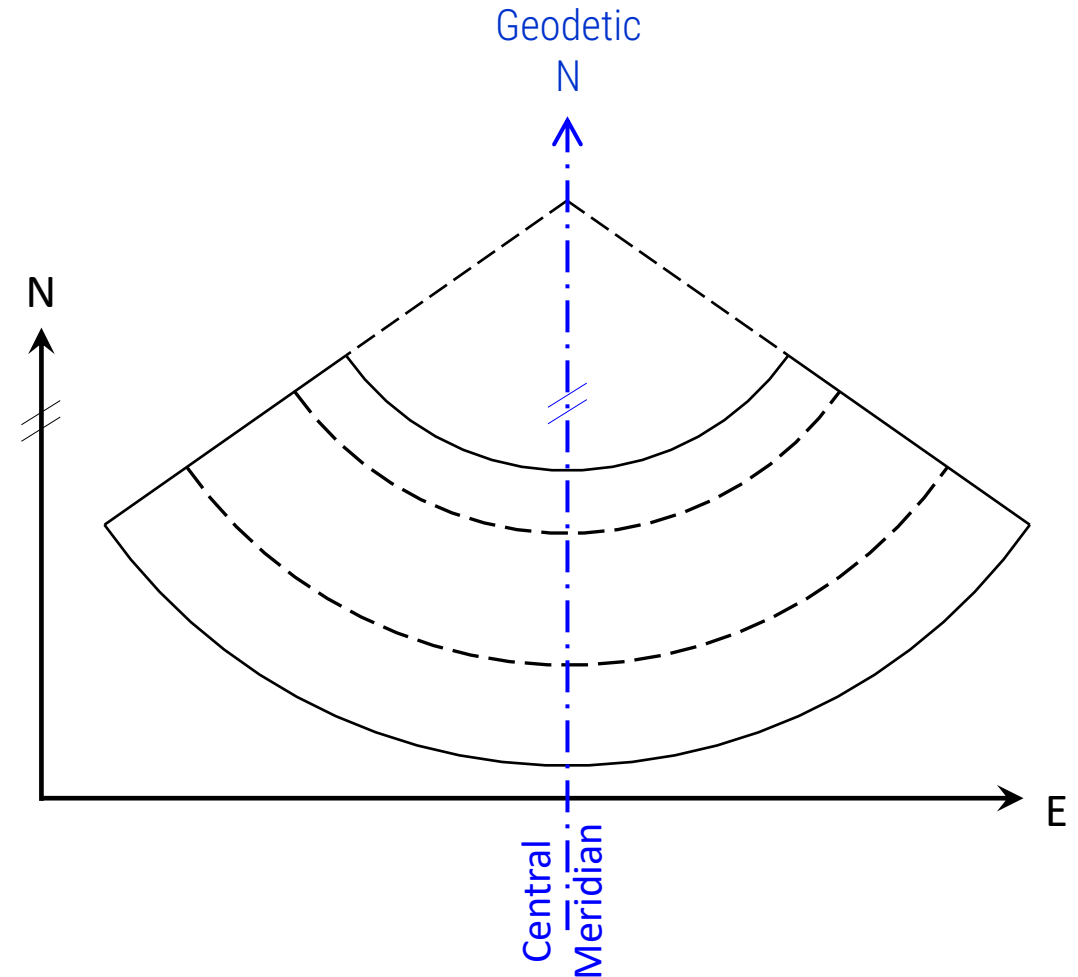
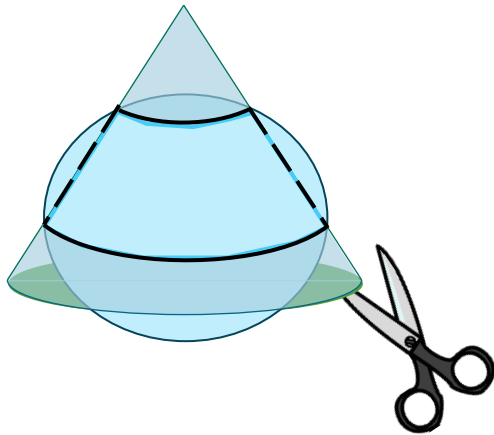


B. Lambert Conic Conformal

Cone placed over (and through)
ellipsoid.

Points projected.

Projection is laid out flat and a
coordinate system overlaid.

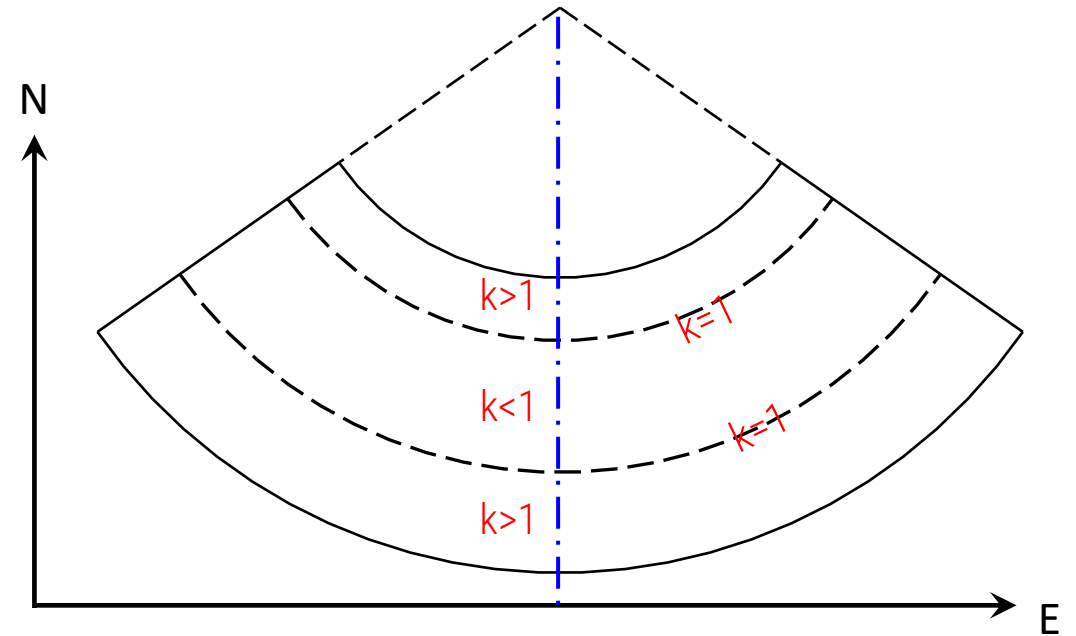
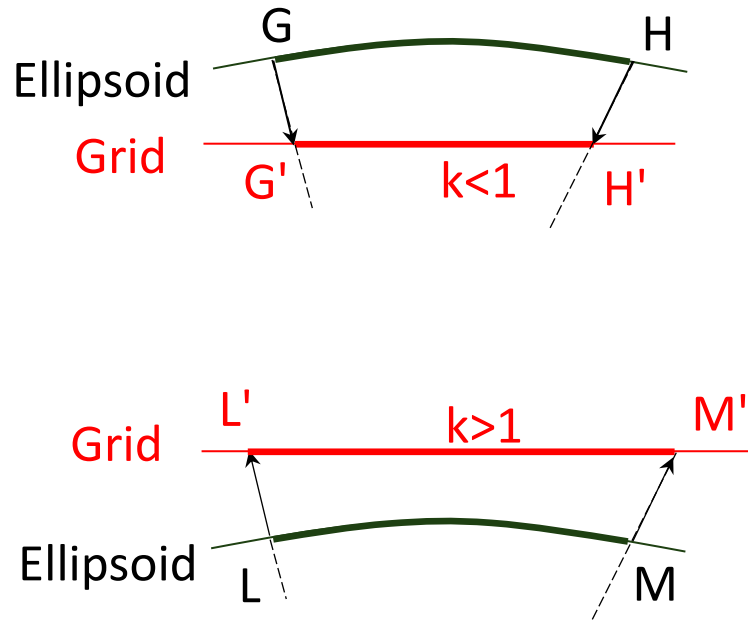


IV. Creating a Grid

B. Lambert Conic Conformal

Distance distortion

Scale, k , is constant E/W; varies N/S



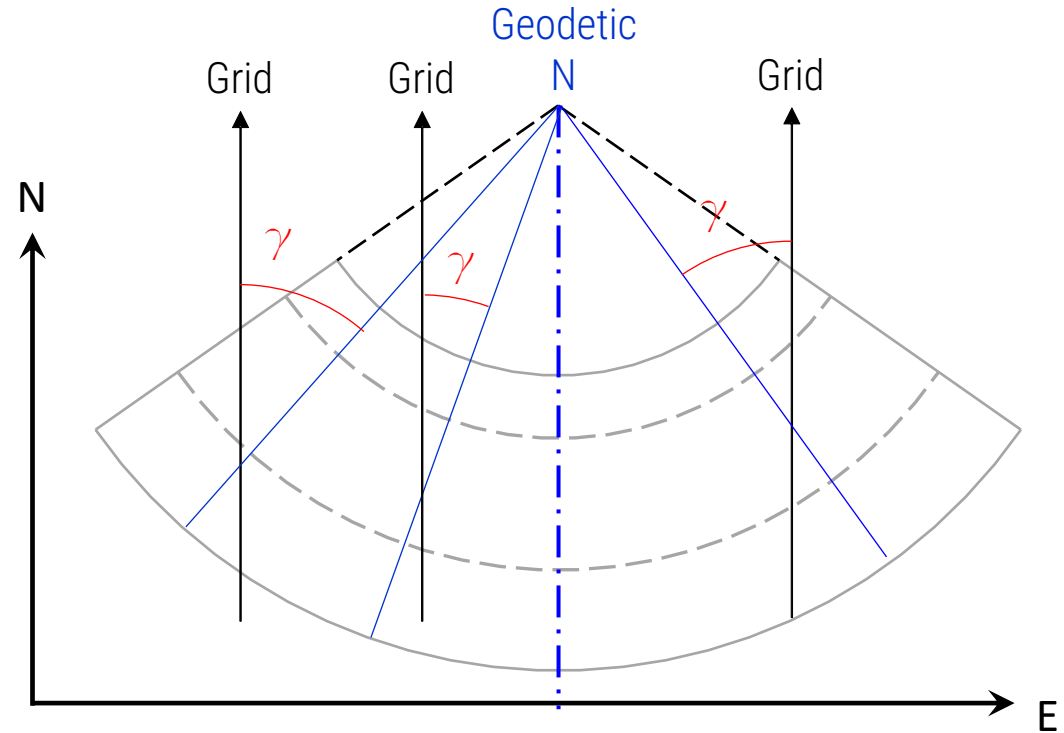
IV. Creating a Grid



B. Lambert Conic Conformal

Direction distortion

Convergence, γ , is angle between Grid and Geodetic North.
 0° at CM, increases to E and to W



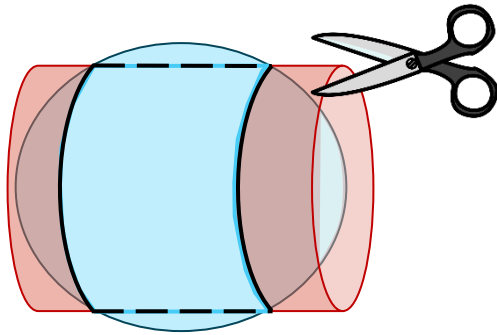
IV. Creating a Grid



C. Mercator Transverse Cylindric

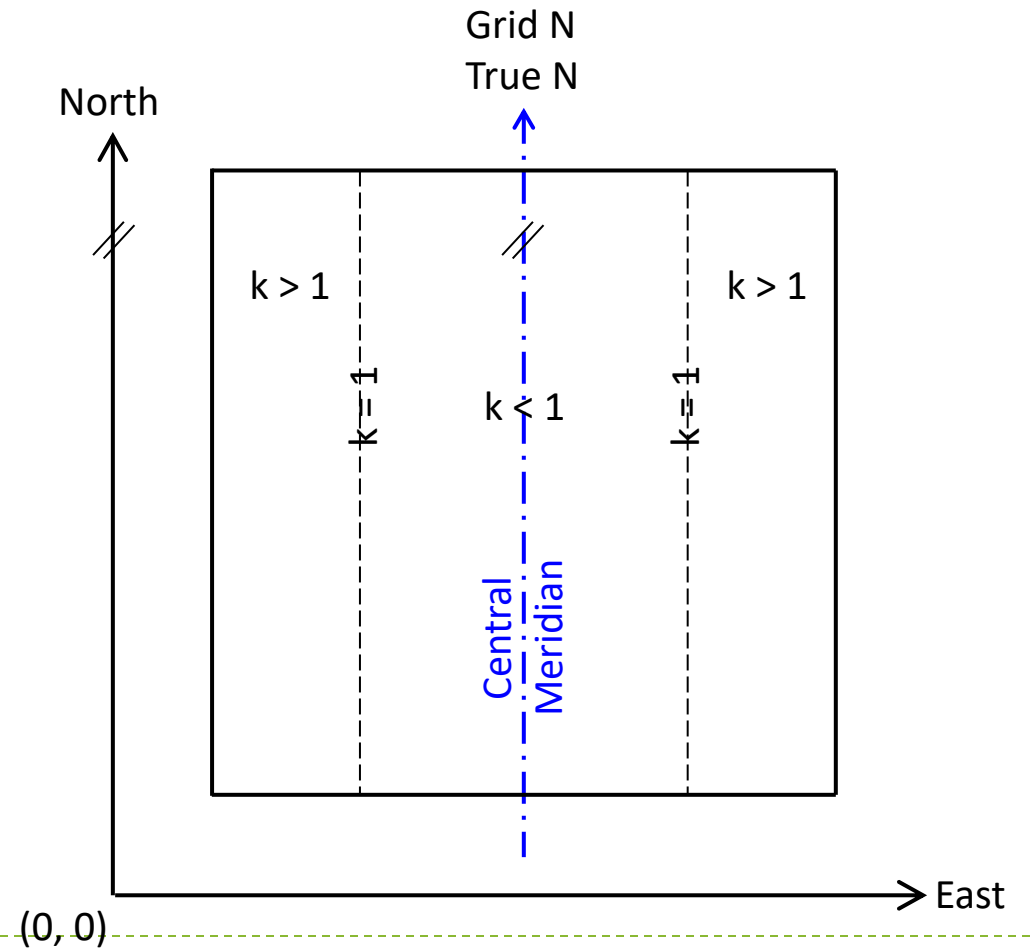
The projection surface is “rolled” out.

Central Meridian defines Grid North



Distance distortion

Scale, k , is constant N/S; varies E/W



IV. Creating a Grid

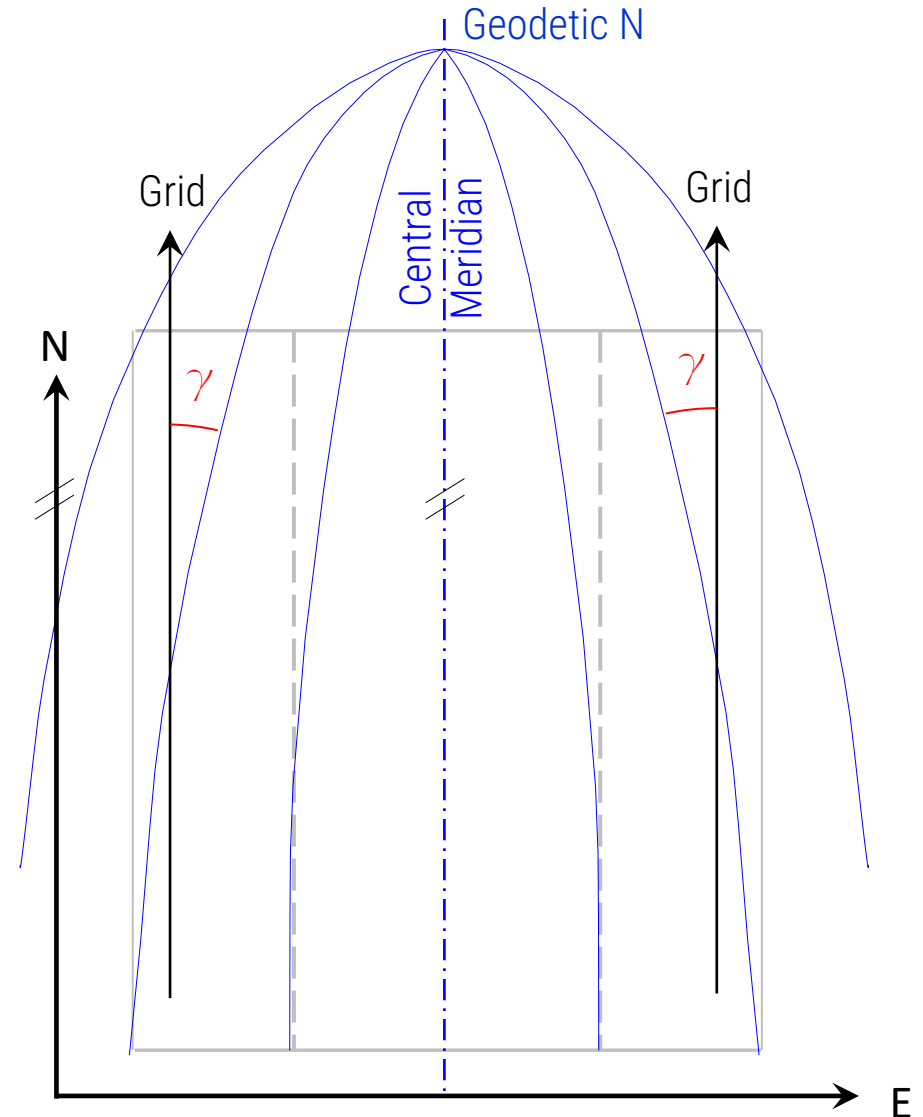


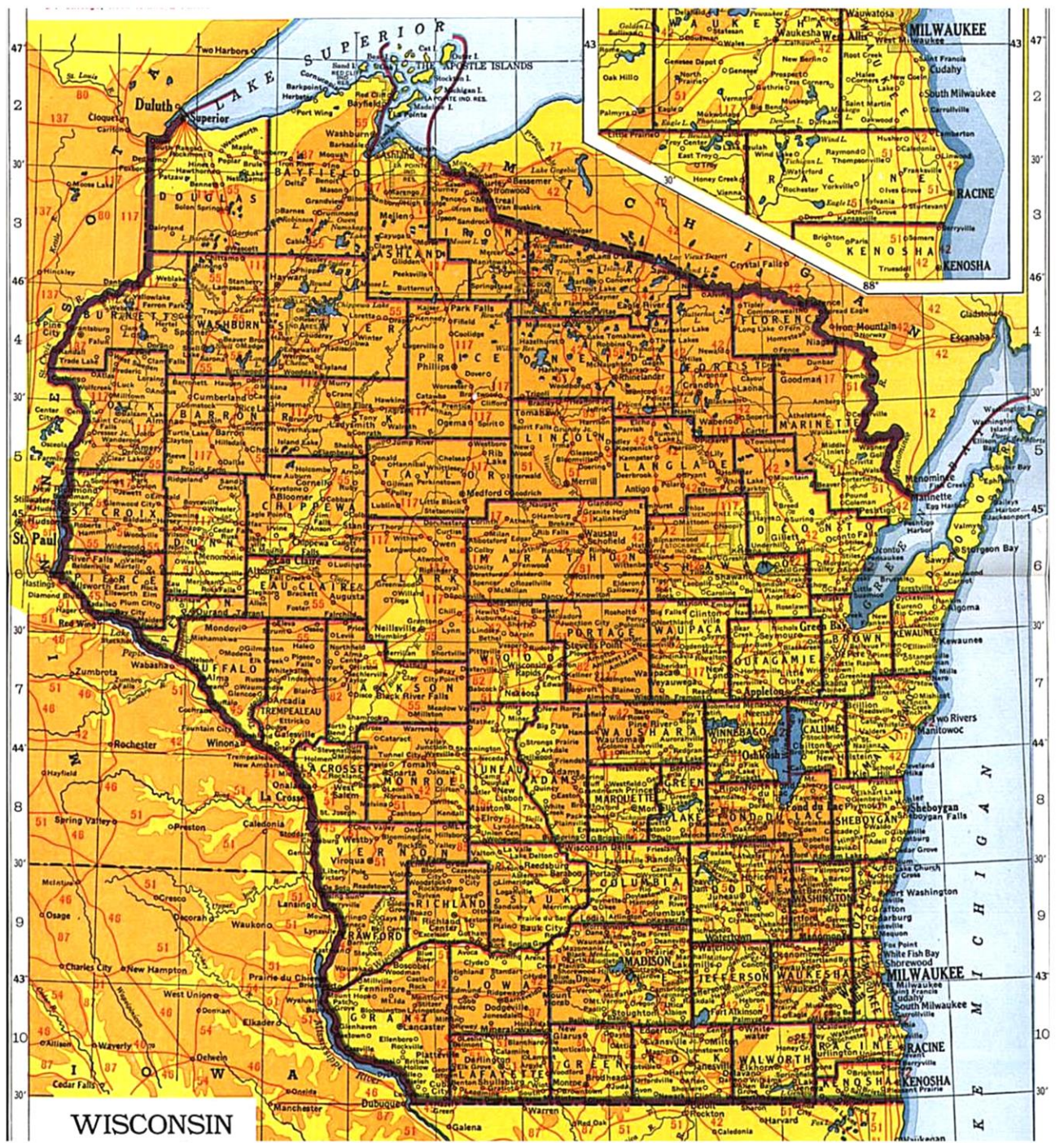
C. Mercator Transverse Cylindric

Direction distortion

Convergence, γ , is angle between Grid and Geodetic North.

0° at CM, increases to E and to W





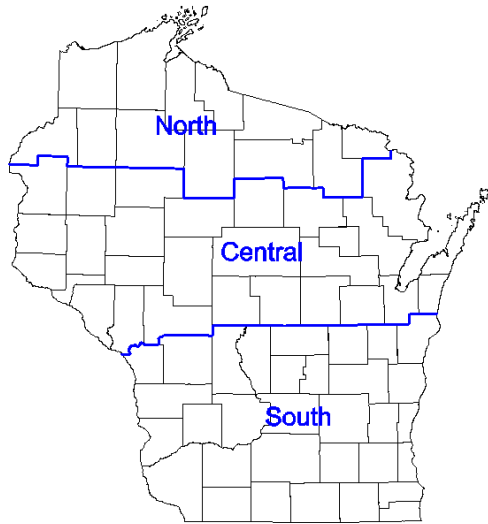
V. Wisconsin Coordinate Systems

V. Wisconsin Coordinate Systems



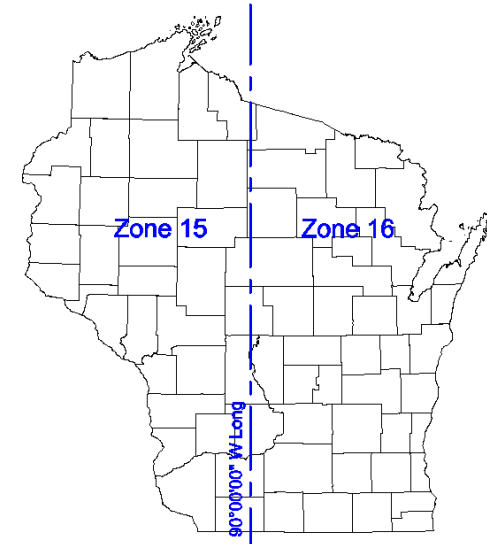
A. Nationally Defined & Supported

1. State Plane Coordinate (SPC) system



Projection	Conic
Zones	3
Max Distortion (ellipsoid to grid)	1/10,000

2. Universal Transverse Mercator (UTM)



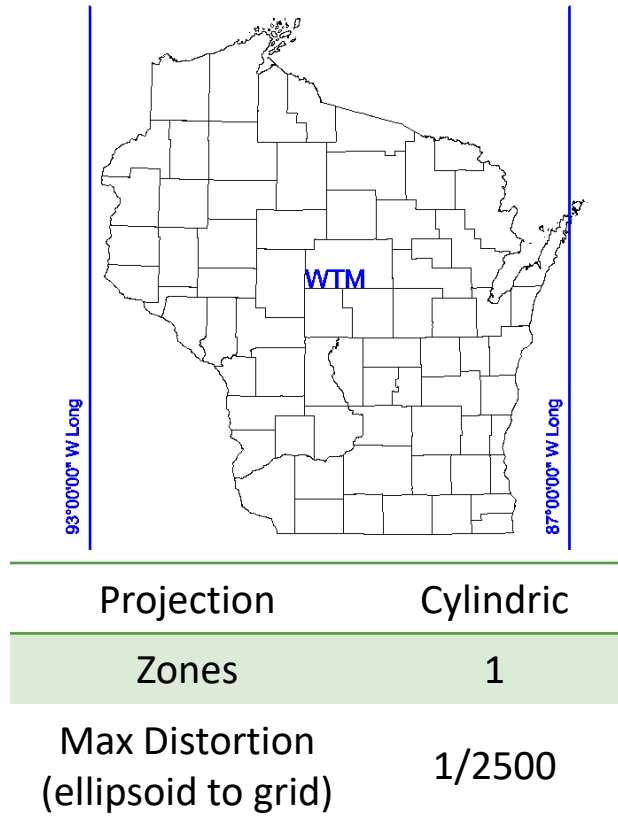
Projection	Cylindric
Zones	2
Max Distortion (ellipsoid to grid)	1/2500

V. Wisconsin Coordinate Systems



B. Locally Defined and Supported

1. Wisconsin Transverse Mercator (WTM) System



Developed by WisDNR to facilitate statewide coverage on a single coord grid.

Modified UTM

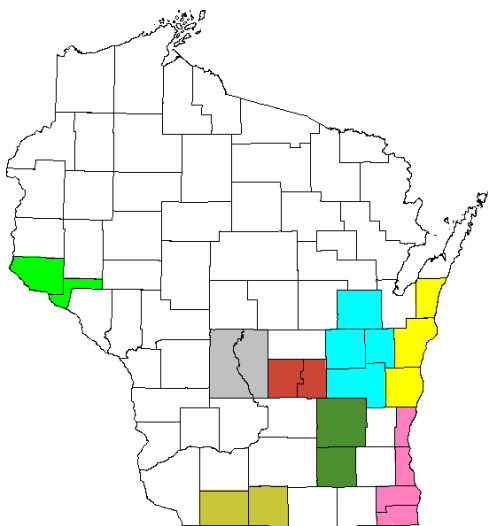
Cylindric projection rotated 3° placing it halfway between UTM Zones 15 & 16.

WisDNR data is generally provided in WTM system.

V. Wisconsin Coordinate Systems

B. Locally Defined and Supported

2. Low Distortion Projections



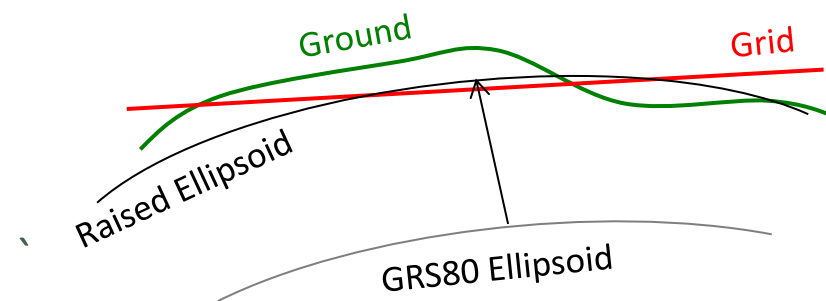
Projection	Cylindric & Conic
Zones	58
Max Distortion (ground to grid)	urban: 1/50,000 rural: 1/30,000

a. WCCS - Wis County Coord Systems

Original design.

All projections used raised ellipsoids

GRS80 + Ave Geoid ht + Median elev



Results in non-standard ellipsoid

⇒ Different datum

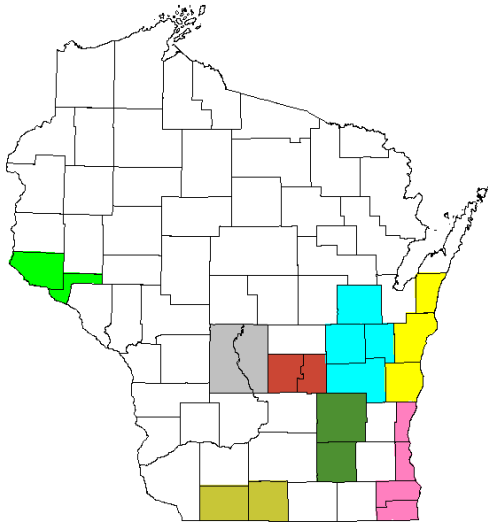
Caused problems with some software.

V. Wisconsin Coordinate Systems



B. Locally Defined and Supported

2. Low Distortion Projections

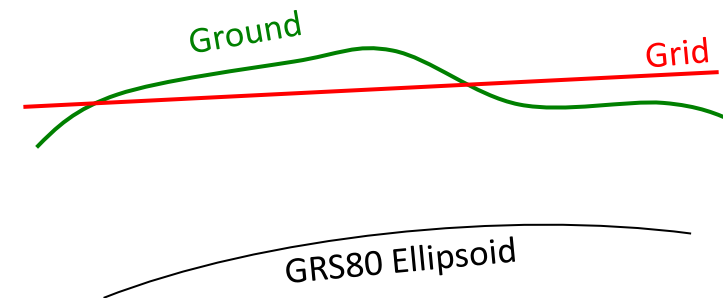


Projection	Cylindric & Conic
Zones	58
Max Distortion (ground to grid)	urban: 1/50,000 rural: 1/30,000

b. WisCRS - Wis Coord Reference Systems

WCCS redefinition

Same zones but each used GRS 80 ellipsoid directly.



Maintains accuracy with respect to WCCS.

Allowable design difference: ± 5 mm



VI. Ground and Grid

What's the beef?

A. Distortion Compensation

1. Distance

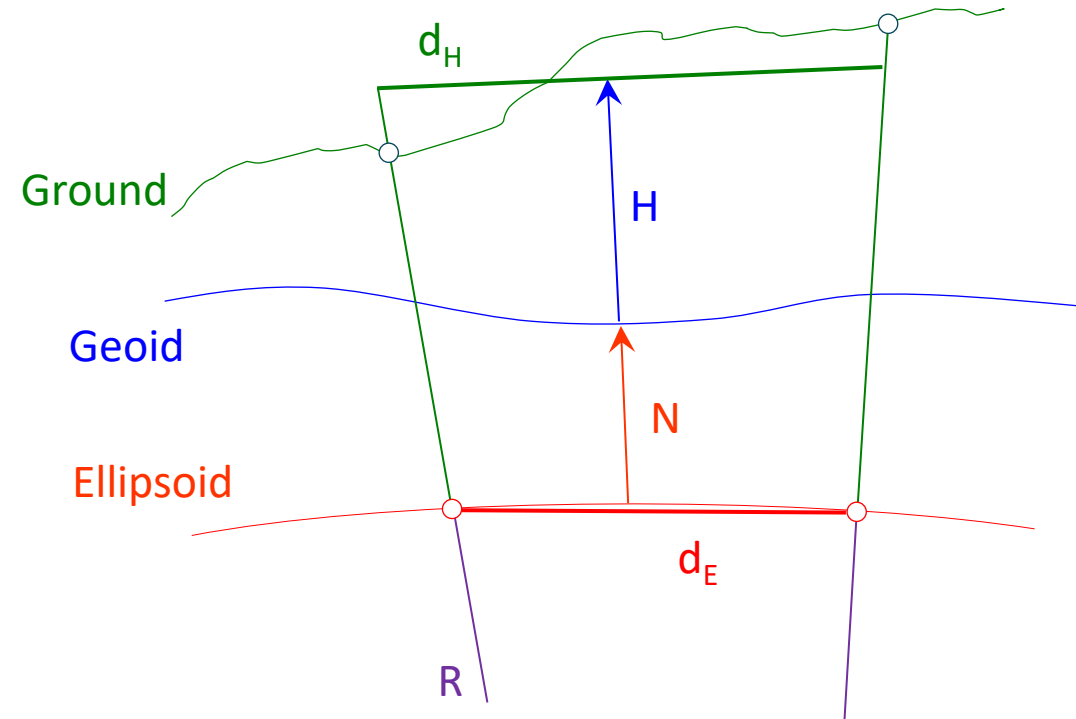
Two steps

a. Ground to ellipsoid

$$EF = \frac{R}{R + H + N}$$

$$d_E = d_H \times EF$$

- d_H Horizontal ground distance
- d_E Ellipsoidal (geodetic) distance
- EF Elevation Factor
- R Mean earth radius
- H Orthometric ht (elev)
- N Geoid height
- k Scale factor



$$R = 20,902,000 \text{ ft} = 6,371,000 \text{ m (approx.)}$$

A. Distortion Compensation

1. Distance

Two steps

b. Ellipsoidal to grid

$$d_G = d_E \times k$$

d_G Grid distance

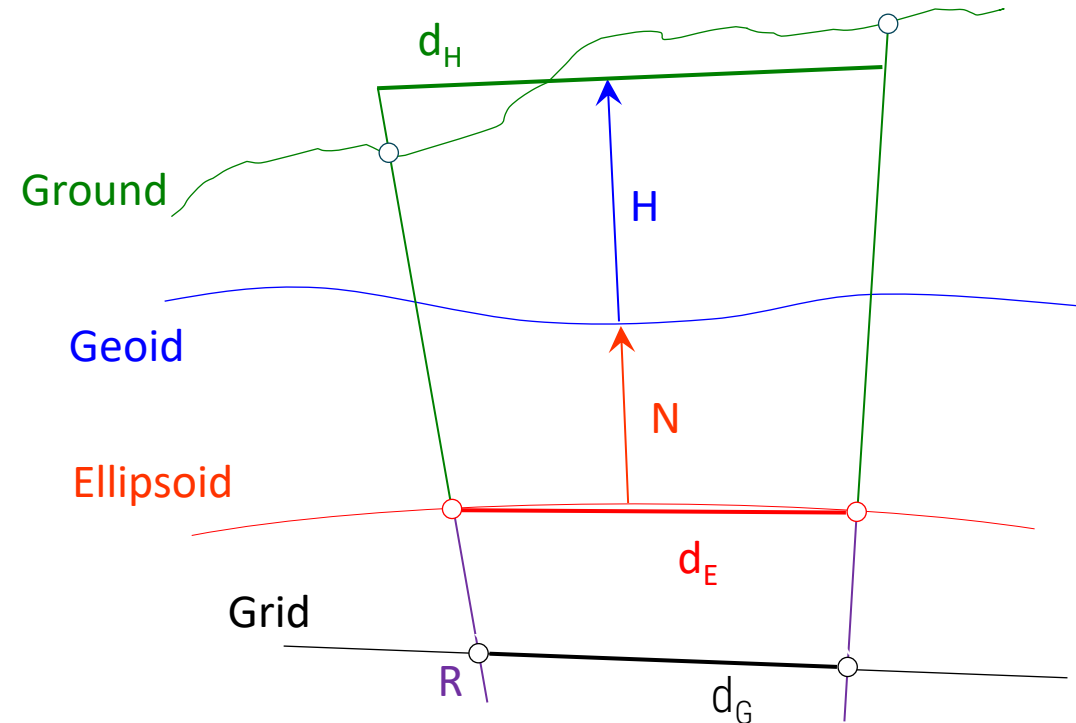
d_E Ellipsoidal (geodetic) distance

k Grid scale factor

c. Combined factor

$$CF = EF \times k$$

$$d_G = d_H \times CF$$



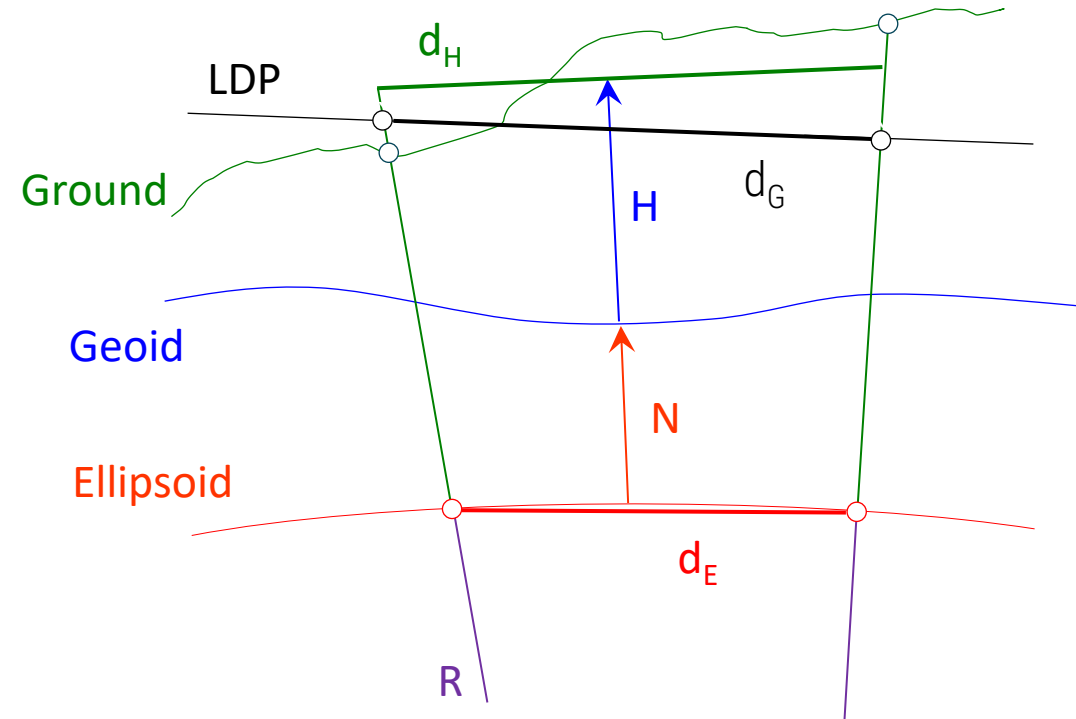
A. Distortion Compensation

1. Distance

c. LDP

Because a LDP grid is near-ground level, there may be no discernible difference between ground and grid distances.

Most of the time, this reduction can be ignored for both WCCS and WisCRS grids.



VI. Ground and Grid

A. Distortion Compensation

2. Direction

The convergence angle, γ , is **from** Geodetic N **to** Grid N

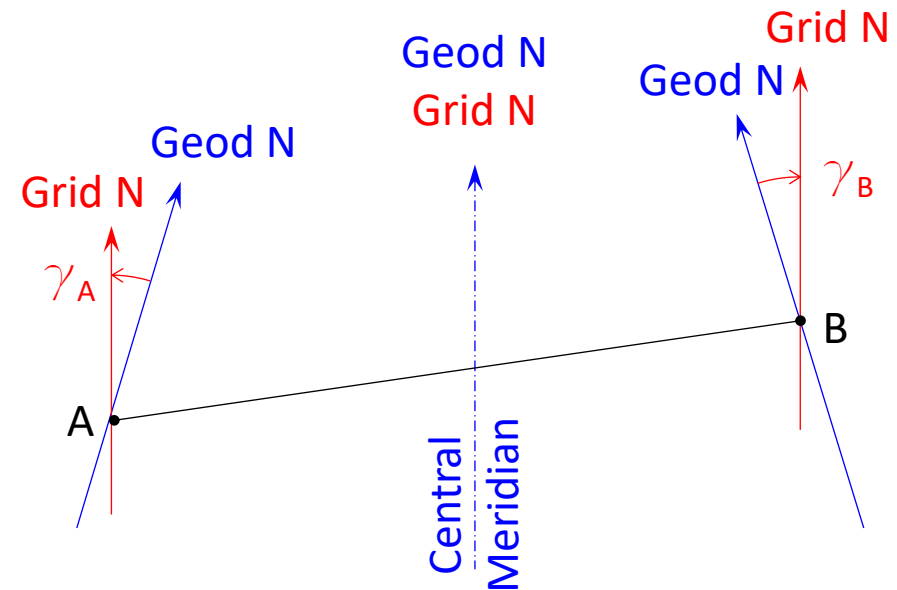
It is positive (cw) East of the CM, negative (ccw) West of the CM

To convert Geodetic (Ground) direction to Grid:

$$t = \alpha - \gamma$$

t	Grid azimuth
α	Geodetic azimuth
γ	Convergence

Might be significant for an LDP



VI. Ground and Grid



B. Reduction Elements

Where do we get the ortho and geoid heights, scale, and convergence angles?

NGS software (*Geodetic Tool Kit*):

*NCAT*¹

GEOIDXX

Ortho heights from USGS topoquads

¹*NCAT* does not currently support local LDPs. When NATRF2022 is adopted, *NCAT* will include NGS-accepted LDPs.

Single Point Conversion | Multipoint Conversion | Web services | Downloads | Tutorial & FAQs | About NCAT

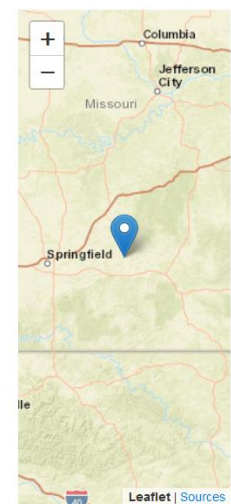
Convert/ Transform from: Horizontal Horizontal+height XYZ

Select the type of horizontal coordinate: Geodetic lat-long SPC UTM USNG

Enter lat-lon in decimal degrees
 Lat
 Lon
 or degrees-minutes-seconds
 Lat
 Lon
 or drag map marker to a location of interest

Input reference frame (historically called 'horizontal datum'):
 Output reference frame (historically called 'horizontal datum'):

Don't see a reference



Click blue bar(s) to expand/collapse

Converted Coordinate							
Reference Frame: NAD83(2011)							
Lat-Lon-Height		SPC		UTM/USNG		XYZ (m)	
Latitude	N42° 49' 42.30000" N424942.30000 42.8284166667	Zone	WI S-4803	Zone	<input type="text" value="16"/>	X	N/A
Longitude	E270° 09' 41.99905" W0895018.00095 -89.8383335972	Northing	92,042.555 (m) 301,976.282 (usft) 301,976.886 (ift)	Northing (m)	4,745,669.139	Y	N/A
Ellipsoid Height ()	Not given	Easting	613,218.813 (m) 2,011,868.721 (usft) 2,011,872.745 (ift)	Easting (m)	268,002.074	Z	N/A
		Convergence (dms)	00 06 39.89	Convergence (dms)	-01 55 49.34		
		Scale factor	0.99998219	Scale factor	1.00026221		
		Combined factor	N/A	Combined factor	N/A		
				USNG	16TBN6800245669		

You may change the default UTM zone. The change is processed interactively once a lat-long is converted; DO NOT click the Submit button.

Customize Export

VI. Ground and Grid



B. Reduction Elements

NSRS Datasheet

```

DESIGNATION - JERRY
PID          - NH0936

* NAD 83(2011) POSITION- 42 54 24.02215(N) 089 43 53.76413(W) ADJUSTED
* NAD 83(2011) ELLIP HT- 324.836 (meters) (06/27/12) ADJUSTED
* NAD 83(2011) EPOCH - 2010.00
* NAVD 88 ORTHO HEIGHT - 358.6 (meters) 1177. (feet) VERTCON

GEOID HEIGHT - -33.902 (meters) GEOID18
NAD 83(2011) X - 21,919.631 (meters) COMP
NAD 83(2011) Y - -4,679,204.603 (meters) COMP
NAD 83(2011) Z - 4,320,134.562 (meters) COMP
LAPLACE CORR - -0.36 (seconds) DEFLEC18

          North          East          Units  Scale Factor  Converg.
SPC WI S - 100,758.292  621,917.891  MT      0.99996957  +0 11 03.9
SPC WI S - 330,571.16  2,040,408.95  sFT     0.99996957  +0 11 03.9
UTM 16 - 4,754,071.382  277,008.712  MT      1.00021177  -1 51 37.6

          - Elev Factor x Scale Factor = Combined Factor
SPC WI S - 0.99994906 x 0.99996957 = 0.99991863
UTM 16 - 0.99994906 x 1.00021177 = 1.00016082
    
```

} At Jerry's elevation

C. Variations

1. Elevation factor, EF

$$EF = \frac{R}{R + H + N}$$

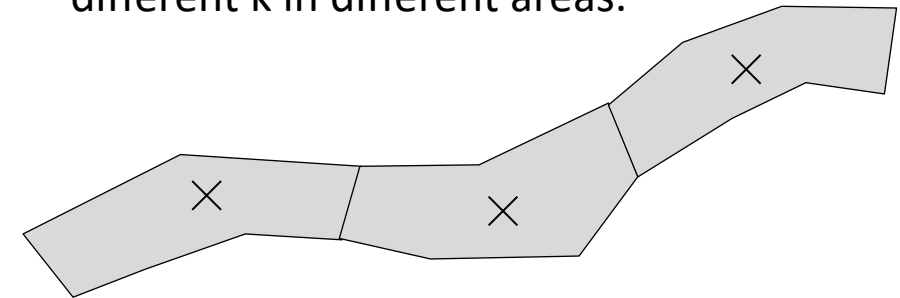
N doesn't change much so can generally use a single value over the project area.

Depending on terrain, H can be:
project area average – use for all lines
computed average for each line

2. Grid scale, k

For relatively small projects, a single value at project center could be used.

Larger/longer projects would require applying different k in different areas.



SPC/UTM - use approx. lat & long with *NCAT* to determine k.

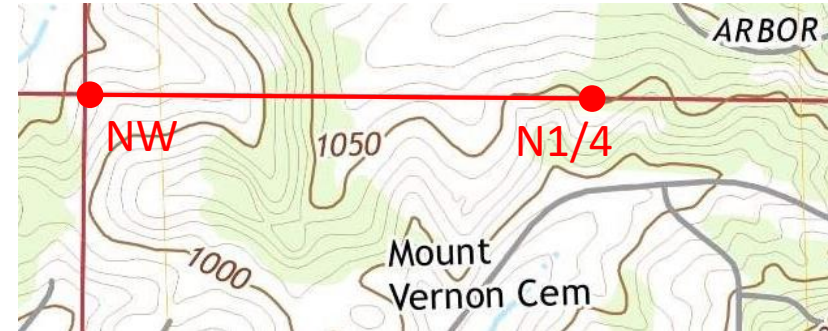
D. SPC Example

Quarter line NW-N1/4 of Sec 34 T5N R7E

Distance: 2638.25 ft

Bearing: S88°11'34"E

Determine WI SPC South zone grid distance
and bearing from NW to N1/4



From topoquad

NW elev: 960 ft

N1/4 elev: 1050 ft

Approx position of NW corner is:

42°57.5' Lat

89°39.75' Long

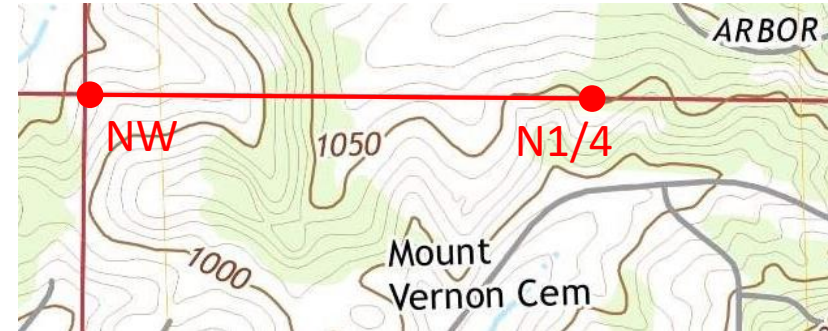
D. SPC Example

Quarter line NW-N1/4 of Sec 34 T5N R7E

Distance: 2638.25 ft

Bearing: S88°11'34"E

Determine WI SPC South zone grid distance and bearing from NW to N1/4



From topoquad

NW elev: 960 ft

N1/4 elev: 1050 ft

Approx position of NW corner is:

42°57.5' Lat

89°39.75' Long

From *NCAT*

$k = 0.99996\ 224$

$\gamma = +0^\circ 13' 53.46''$

From *GEOID18*

$N = -34.046\ m$

D. SPC Example

1. Distance

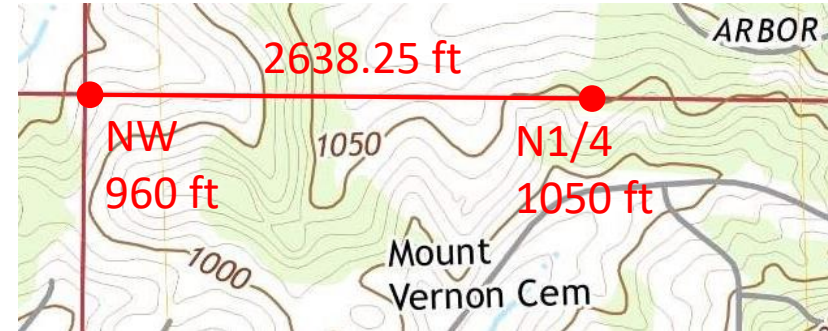
a. Ground to ellipsoid

$$H = \frac{960 + 1050}{2} = 1005$$

$$EF = \frac{20,902,000}{20,902,000 + 1005 + (-111.7)}$$

$$= 0.999957264$$

$$d_E = 2638.25 \text{ ft} \times 0.999957264 = 2638.137 \text{ ft}$$



$$R = 20,902,000 \text{ ft}$$

From *NCAT*

$$k = 0.99996224$$

$$\gamma = +0^\circ 13' 53.46''$$

From *GEOID18*

$$N = -34.046 \text{ m} = -111.7 \text{ ft}$$

D. SPC Example

1. Distance

a. Ground to ellipsoid

$$H = \frac{960 + 1050}{2} = 1005$$

$$EF = \frac{20,902,000}{20,902,000 + 1005 + (-111.7)}$$

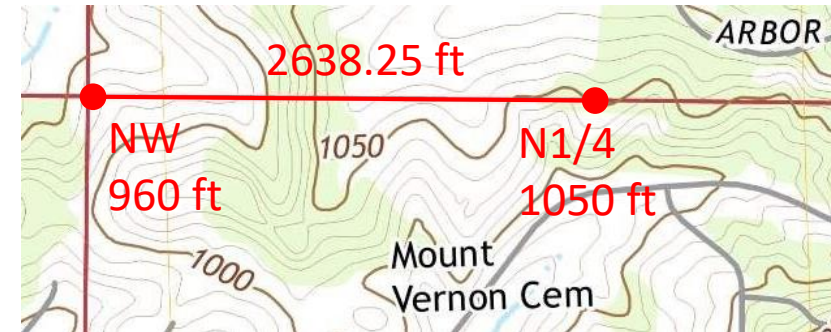
$$= 0.99995\ 7264$$

$$d_E = 2638.25\text{ft} \times 0.99995\ 7264 = 2638.137\text{ft}$$

b. Ellipsoid to grid

$$d_G = d_E \times k$$

$$d_G = 2638.137\text{ft} \times 0.99995\ 7264 = \underline{2638.024\text{ft}}$$



$$R = 20,902,000\text{ ft}$$

From *NCAT*

$$k = \mathbf{0.99996\ 224}$$

$$\gamma = +0^\circ 13' 53.46''$$

From *GEOID18*

$$N = -34.046\text{ m} = -111.7\text{ ft}$$

D. SPC Example

2. Direction

Convert bearing to azimuth

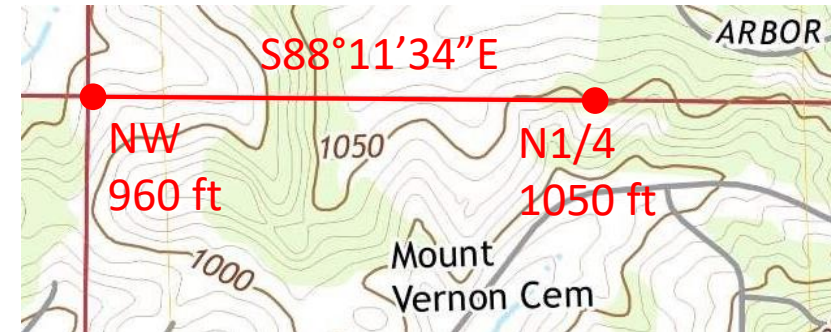
$$\begin{aligned} Az &= 180^{\circ}00'00'' - 88^{\circ}11'34'' \\ &= 91^{\circ}48'26'' \end{aligned}$$

Convert to Grid Az

$$\begin{aligned} \text{Grid Az} &= 91^{\circ}48'26'' - (+0^{\circ}13'53'') \\ &= 92^{\circ}02'19'' \end{aligned}$$

Convert to bearing

$$\begin{aligned} \text{Grid Brg} &= 180^{\circ}00'00'' - 92^{\circ}02'19'' \\ &= \underline{S87^{\circ}17'41''E} \end{aligned}$$



$$R = 20,902,000 \text{ ft}$$

From *NCAT*

$$k = 0.99996 \ 224$$

$$\gamma = +0^{\circ}13'53.46''$$

From *GEOID18*

$$N = -34.046 \text{ m} = -111.7 \text{ ft}$$

E. WCCS/WisCRS

1. Reduction Elements

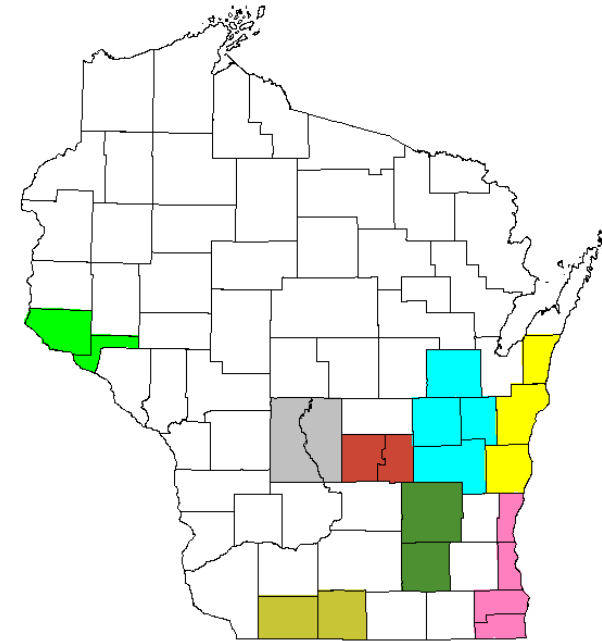
LDPs are not supported by *NCAT*, how to get N, convergence, and scale?

N: Use *GEOIDXX*, but need geodetic coordinates.

γ and k: ?

Need LDP-specific software.

Some surveying and mapping software have support for LDPs, including Wisconsin's.



VI. Ground and Grid



E. WCCS/WisCRS

2. ConCoord v0.95

At <https://jerrymahun.com>

The screenshot shows the ConCoord v0.95 application window. It features two columns of settings for 'From System' and 'To System', both set to 'County - WisCRS' with 'Iowa' selected in the county dropdown and 'Survey ft' in the units dropdown. The 'Input/Output' is set to 'Manual Entry'. Below these are input fields for 'Convert...' (North: 105692.910, East: 457593.430) and 'Results...' (North: 105,692.9100 sft, East: 457,593.4300 sft, Convergence: +0°13'09.94", Scale: 1.00004 80624). Action buttons for 'Convert', 'Reset', 'Help', and 'Quit' are on the right. A footer note reads 'J. Mahun Feb 2021'.

VI. Ground and Grid



E. WCCS/WisCRS

3. Wis83CoordConv1-5.xlsm

At <https://jerrymahun.com>

	A	B	C	D	E	F	G	H	I	J
1	Wisconsin NAD 83 Coordinate Conversions									
2	Version 1.5 - 4/24									
3		Select units from list								
4										
5		Units	Survey Ft	Enter values to convert in the colored cells						
6				Lat (DD.MMSSss)	Long (DD.MMSSss)	North (Survey Ft)	East (Survey Ft)	Convergence	Grid Scale	
7	State Plane	South		43° .19'.43.5"	90° .27'.37."	484,625.7693	1,846,032.7115	-0°18'58.530"	0.99993	33442
8				+42°48'42.093525"	+88°23'30.583583"	300,000.0000	2,400,000.0000	+1°06'17.927"	0.99998	51232
9										
10	UTM / WTM	WTM		43° .19'.43.5"	90° .27'.37."	1,041,421.2355	1,583,606.1704	-0°18'57.019"	0.99961	71285
11				+41°15'22.123365"	+94°27'01.262190"	317,427.0000	482,683.0000	-2°56'16.999"	1.00131	15573
12										
13	WCCS Conic	Richland		43° .19'.43.5"	90° .27'.37."	442,817.4664	656,090.8685	-0°01'13.413"	0.99999	50587
14	See Map			+42°39'12.042340"	+91°52'35.248029"	200,000.0000	275,000.0000	-0°59'31.330"	1.00006	27240
15										
16	WCCS Cylindric	Polk		45° .14'.00."	92° .13'.00."	208,919.2568	572,350.9903	+0°17'44.980"	1.00001	31574
17	See Map			+44°50'26.723394"	+92°10'52.891669"	65,825.6480	582,246.7690	+0°19'07.349"	1.00001	56964
18										
19	WisCRS Conic	Richland		43° .19'.43.5"	90° .27'.37."	442,817.4674	656,090.8678	-0°01'13.413"	1.00003	75716
20	See Map			+42°25'42.432849"	+91°57'27.067401"	118,430.0705	251,695.2414	-1°02'51.547"	1.00015	82369
21										
22	WisCRS Cylindric	Iowa		45° .14'.00."	92° .13'.00."	989,000.2534	-158,595.4739	-1°27'34.985"	1.00035	97454
23	See Map			+42°49'42.325888"	+89°50'18.002180"	105,692.9100	457,593.4300	+0°13'09.937"	1.00004	80624
24										
25										

Instructions

WCCS or WisCRS?

References

This workbook **DOES NOT** convert coordinates between datum realizations

F. WisCRS Example

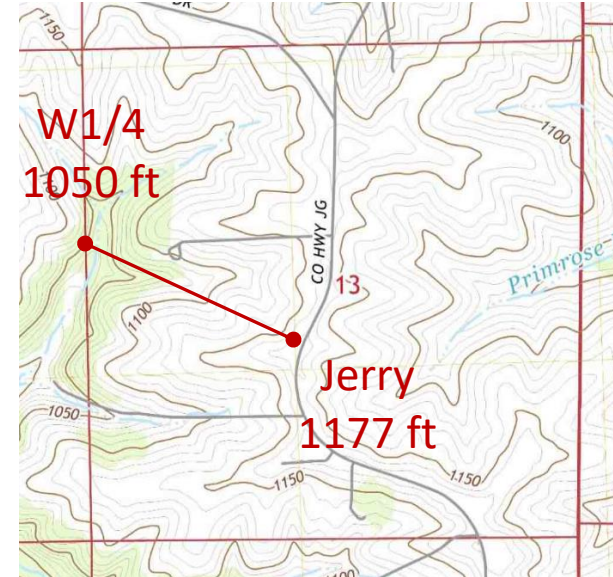
NSRS point *Jerry* to W1/4 cor Sec 13 T5N R6E

Distance: 2313.88 ft

Azimuth: 283°39'44"

Elev: 1050 ft (topoquad)

What are the distance and direction in the
WisCRS Dane County Coord System?



Jerry's data sheet has H and N, but not Dane Co
coords data.

Must compute needed elements.

Use *Jerry's* Lat and Long in *ConCoord*.

Pt *Jerry*

42°54'24.02215" Lat

89°43'53.76413" Long

H = 1177 ft

N = -33.902 m = 111.2 ft

VI. Ground and Grid



F. WisCRS Example

ConCoord v0.95

From System: Geodetic

To System: County - WisCRS

Input/Output: Manual Entry

Select County: Dane

Parameters

Units: Survey ft

Convert...

Latitude: 42.542402215

Longitude: 89.435376413

Format: DDD.MMSSsssss

Results...

North: 421,741.9099 sft

East: 728,105.7413 sft

Convergence: -0°12'40.57"

Scale: 1.00004 24995

Convert

Reset

Help

Quit

J. Mahun Feb 2021

F. WisCRS Example

1. Distance

$$H = \frac{1177 + 1050}{2} = 1113.5$$

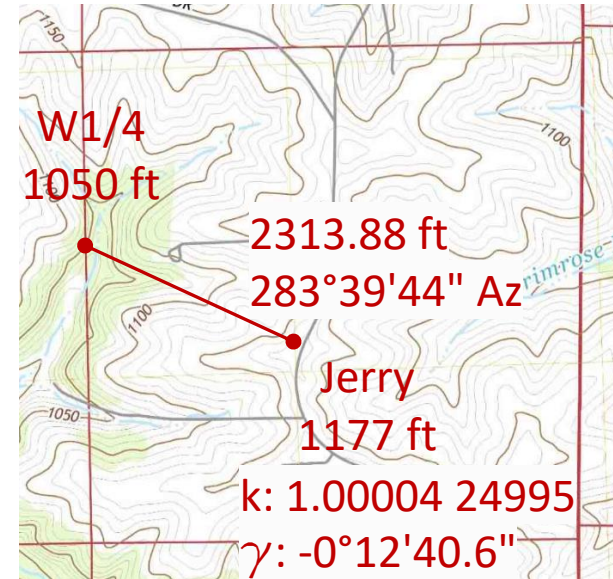
$$EF = \frac{20902000}{20902000 + 1113.5 + (-111.2)}$$
$$= 0.999952050$$

$$CF = 0.999952050 \times 1.0000424995$$
$$= 0.999994547$$

$$d_G = 2313.88 \text{ ft} \times 0.999994547 = \underline{2313.867 \text{ ft}}$$

2. Direction

$$\text{Grid Az} = 283^\circ 39' 44'' - (-0^\circ 12' 40.6'')$$
$$= \underline{283^\circ 52' 24.6''}$$



F. WisCRS Example

1. Distance

$$H = \frac{1177 + 1050}{2} = 1113.5$$

$$EF = \frac{20902000}{20902000 + 1113.5 + (-111.2)}$$

$$= 0.999952050$$

$$CF = 0.999952050 \times 1.0000424995$$

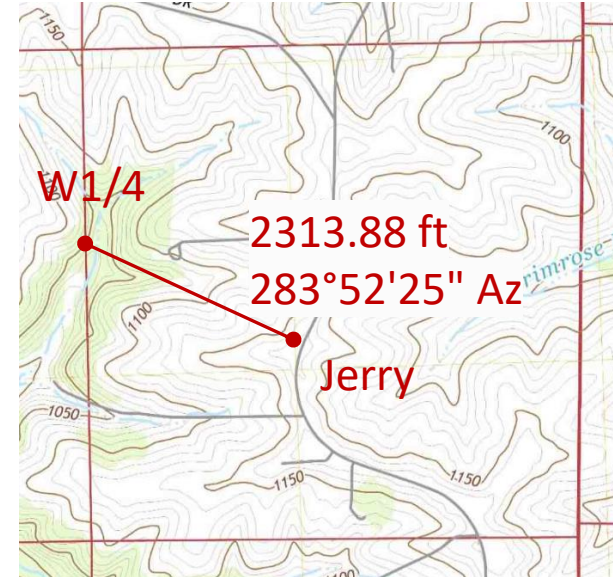
$$= 0.999994547$$

$$d_G = 2313.88ft \times 0.999994547 = \underline{2313.867ft}$$

2. Direction

$$Grid\ Az = 283^\circ 39' 44'' - (-0^\circ 12' 40.6'')$$

$$= \underline{283^\circ 52' 24.6''}$$

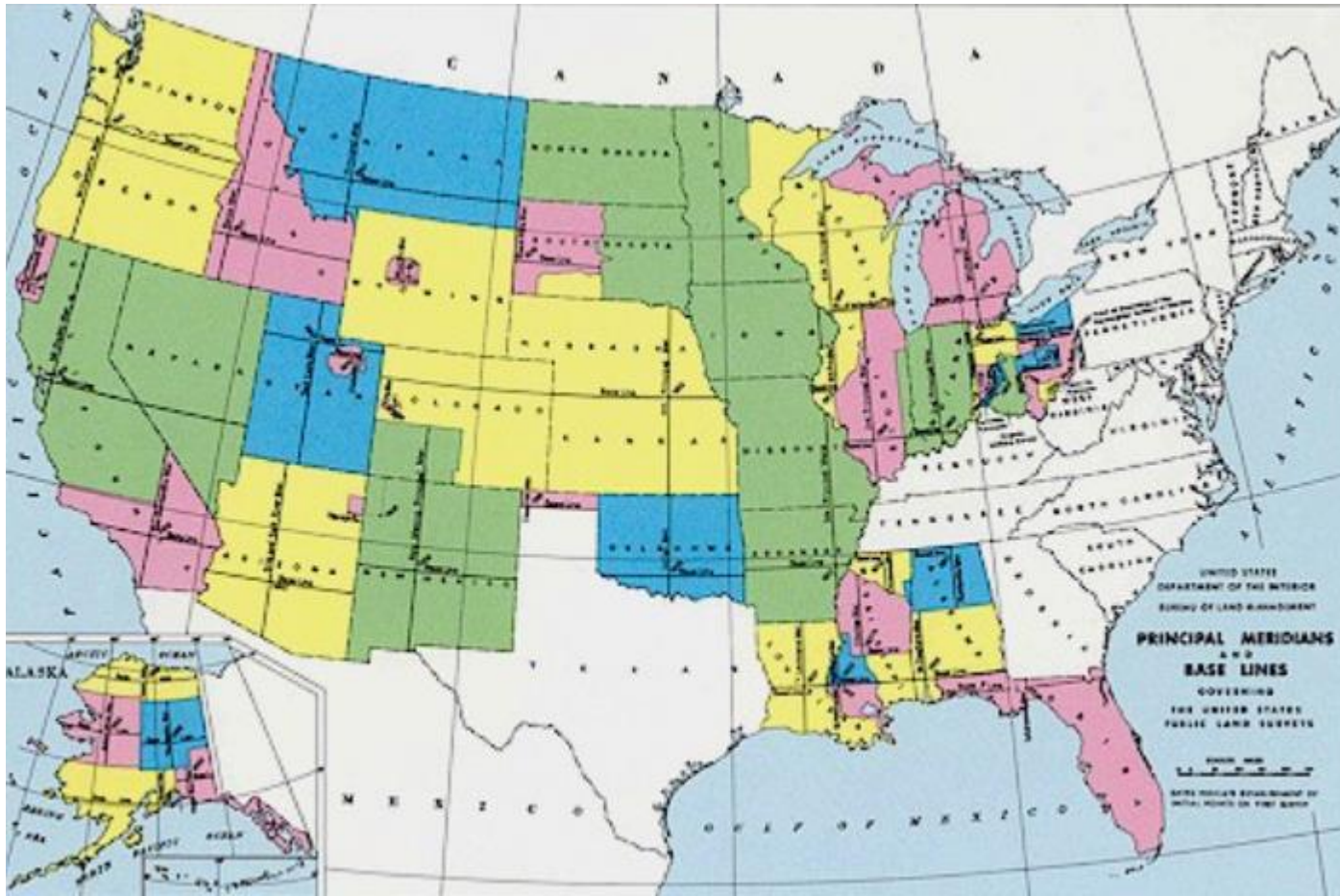


Distance distortion?

$$\frac{1}{1 - CF} = \frac{1}{1 - 0.999994547} = 183385$$

$$\Rightarrow \frac{1}{183,400}$$

Can ignore reduction

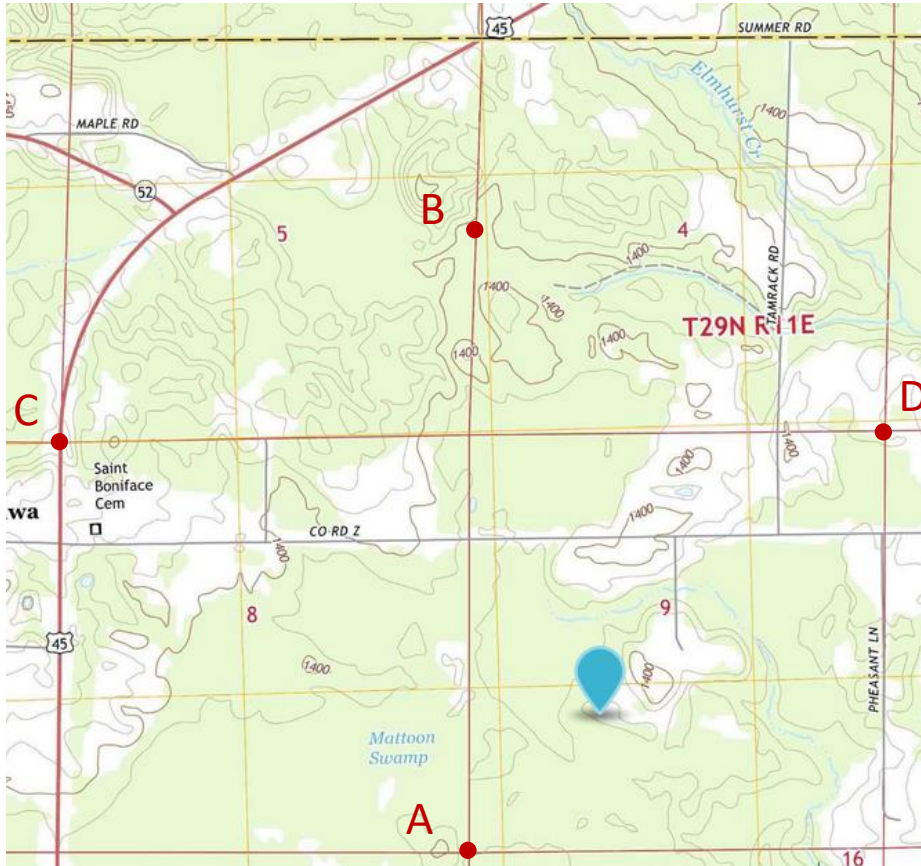


VII. Grids and PLSS Lost Corners

VII. Grids and PLSS Lost Corners



A. Cardinal Equivalents



Proportionate Measurement

Recreating original locations based on record and contemporary measurements.

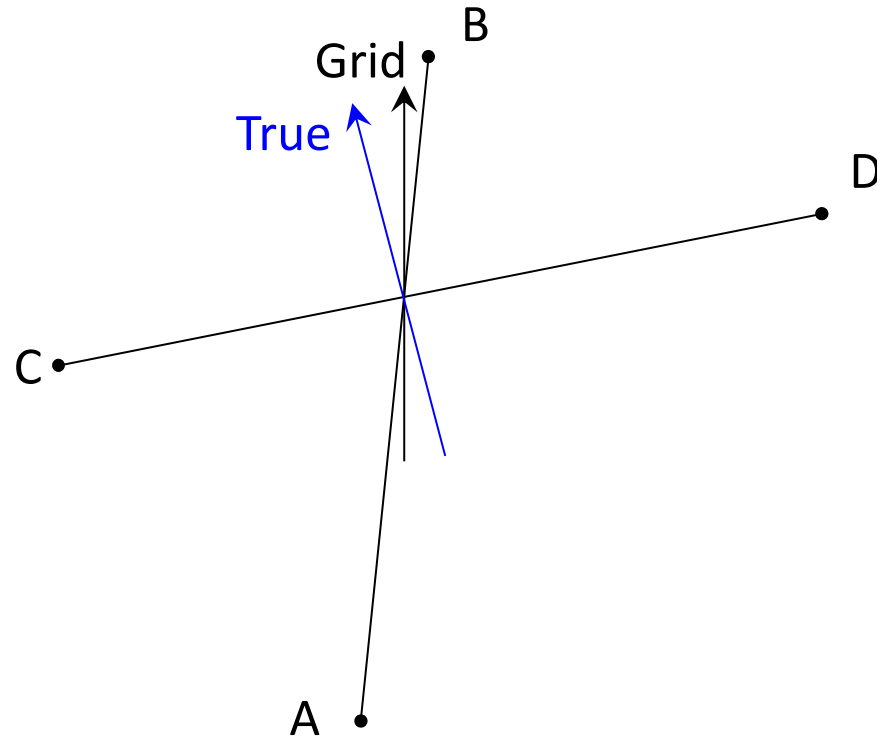
Proportioning.

PLSS Manual states that proportioning must be done in *cardinal directions*

True N/S & True E/W

(*2009 Manual* treats Geodetic and True the same, which isn't technically correct, but close enough.)

A. Cardinal Equivalents



Proportionate Measurement

If working in a grid system must compensate for convergence.

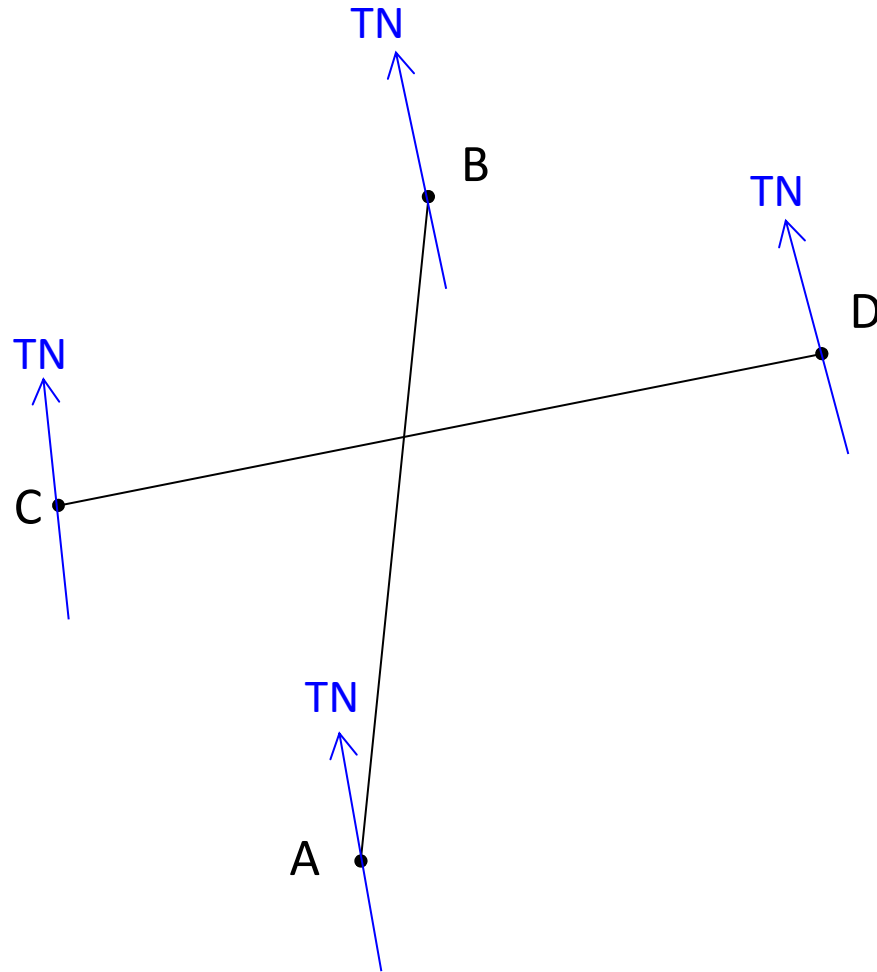
Before or after proportioning?

How?

$\gamma = f(\text{Longitude})$ - it's not constant along E/W lines

Is its effect significant?

B. Double Proportionate Procedure



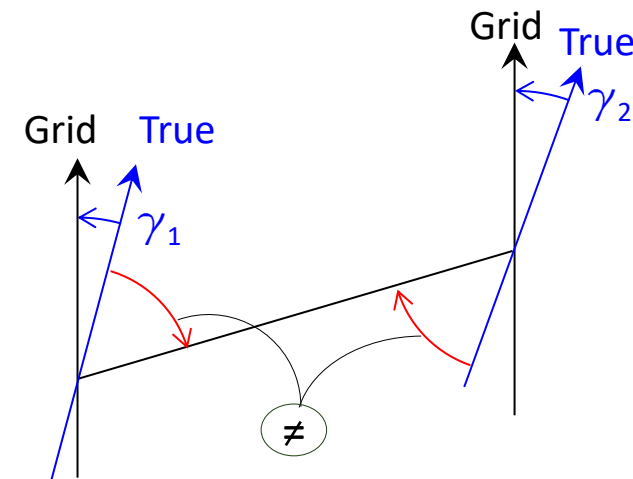
1. Compensate Before Proportioning

Compute grid bearings from coordinates

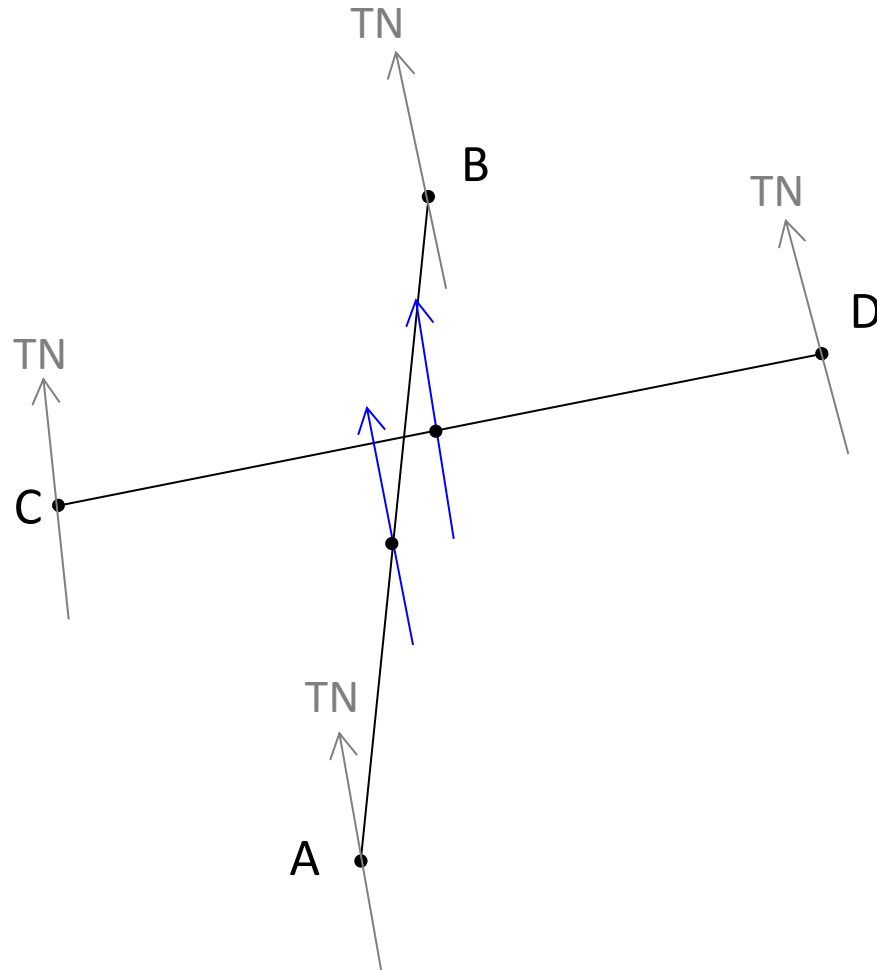
Determine convergence at each point

Convert grid to true directions

True directions are not exactly 180° apart



B. Double Proportionate Procedure



1. Compensate Before Proportioning

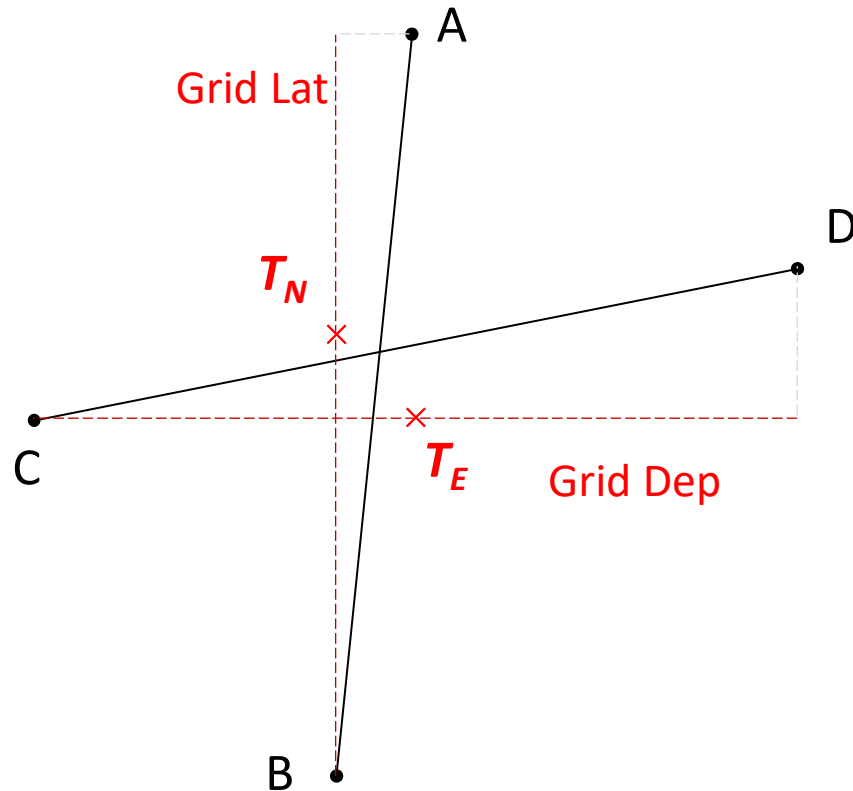
PLSS Manual: use mean bearing of a line for its cardinal computations.

Compute the true bearings at line midpoints.

Then compute cardinal equivalents.

Continue regular DPM process.

B. Double Proportionate Procedure



2. Compensate After Proportioning

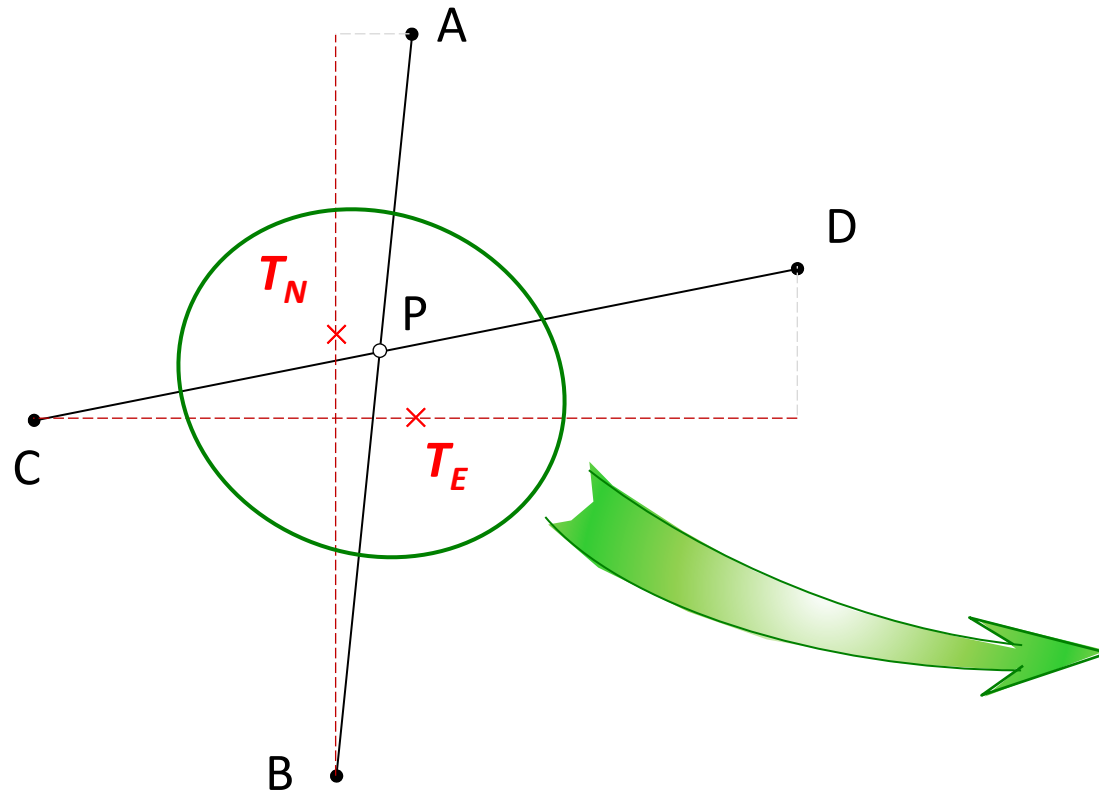
Set temporary points using grid equivalents

N-S	Grid Brng _{AB}	Grid Dist _{AB}
	Grid Lat _{AB}	Grid Dep _{AB}
	Set T _N by SPM	
E-W	Grid Brng _{CD}	Grid Dist _{CD}
	Grid Lat _{AB}	Grid Dep _{AB}
	Set T _E by SPM	

Compute γ_N and γ_E at the temp points.

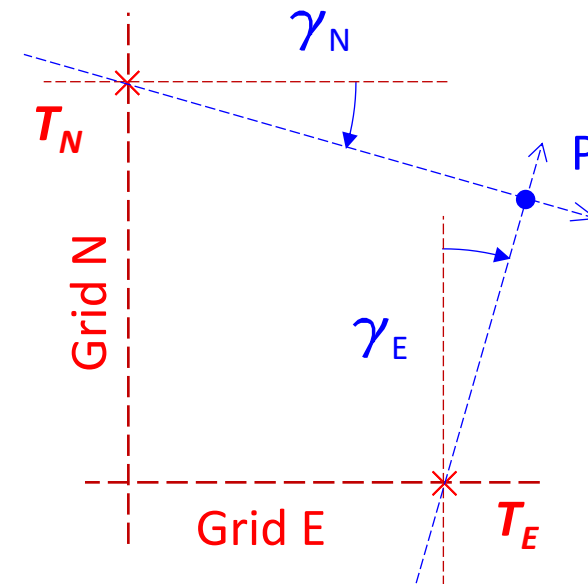
VII. Grids and PLSS Lost Corners

B. Double Proportionate Procedure

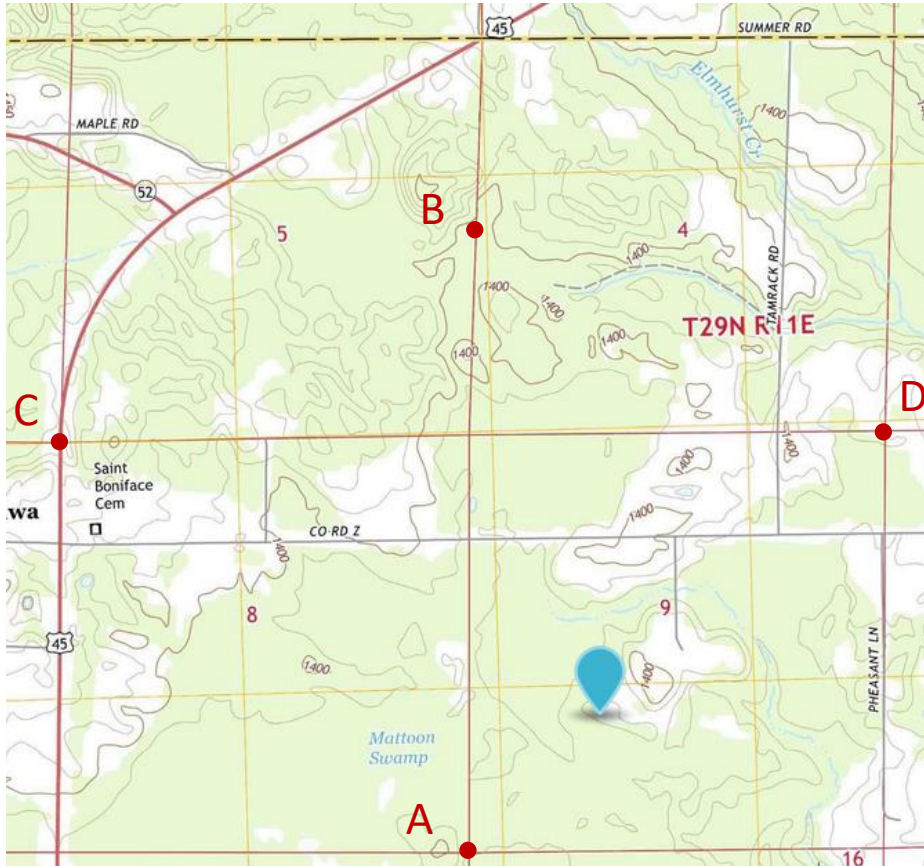


2. Compensate After Proportioning

At the two temporary points project lines rotated γ_N and γ_E from Grid N and Grid E. Set lost corner at intersection of the lines.



C. Significant Effect?



Does ignoring convergence affect corner position?

Example: Shawano County

SW cor S4 T29N R11E is lost.

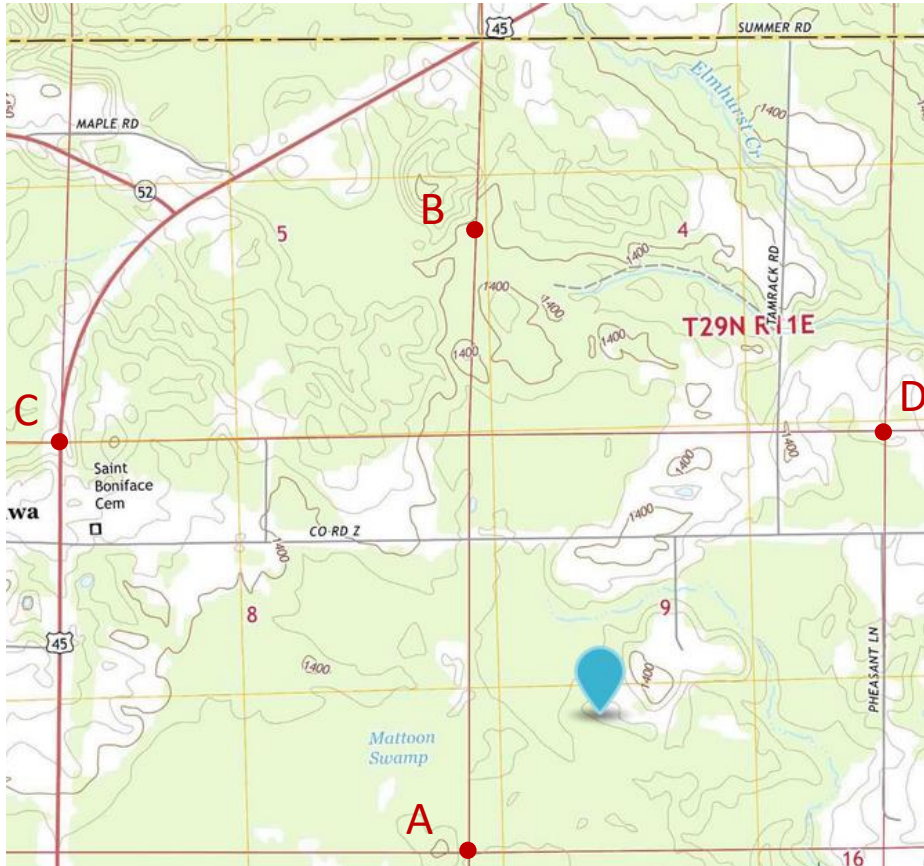
Existing corners and their Shawano Co WisCRS coordinates are:

ID	Corner	North (ft)	East (ft)	γ
A	SW S9	352,289.81	711,348.81	-0°24'32"
B	W1/4 S4	360,188.90	711,493.25	-0°24'33"
C	SW S5	357,550.43	706,134.07	-0°25'25"
D	SE S4	357,607.73	716,672.60	-0°23'46"

γ computed using ConCoord,

VII. Grids and PLSS Lost Corners

C. Significant Effect?



Does convergence have a significant effect?

Computed values

<i>Pt</i>	<i>North (ft)</i>	<i>East (ft)</i>	γ
T _N	357,556.08	711,348.74	-0°24'33"
T _E	357,550.43	711,403.34	-0°24'33"

<i>Pt</i>	<i>North (ft)</i>	<i>East (ft)</i>	<i>Convergence</i>
SW4	357,556.08	711,403.34	Not applied
SW4	357,555.69	711,403.37	Applied

Coordinates are close, but the two positions are 0.39 ft apart.

Enough for a pin cushion.

I. Spatial Systems

II. Distortions

III. Earth Models

IV. Creating a Grid

V. Wisconsin Coordinate Systems

VI. Ground and Grid

VII. Grids and PLSS Lost Corners

Grid ↔ Ground - Simple, Right?

