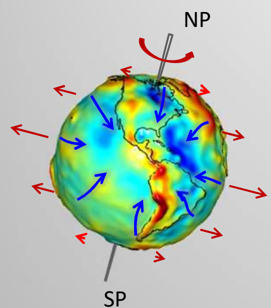
WESTERN REGIONAL SURVEY CONFERENCE
MARCH 18 - 21, 2026

Ground and Grid
Simple, Right?



Thu 19 Mar 2026

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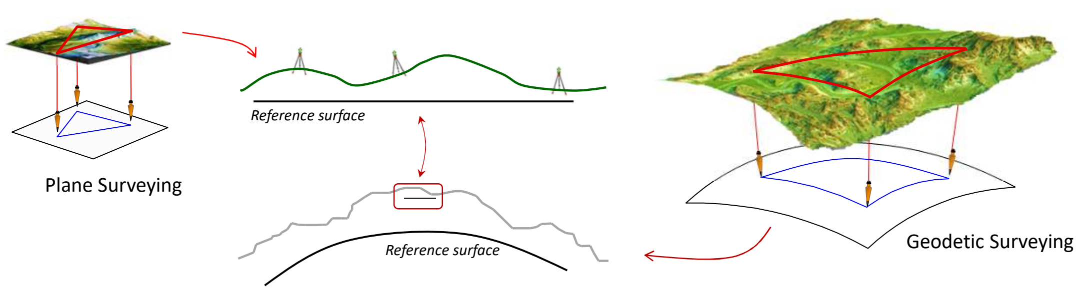
Grid and Ground
Simple, Right?

I. Earth Models

I. Earth Models

A. Physical Earth - Ground

- The surface on which we measure: Not mathematical.
- Plane Surveying: Small areas, can assume a flat reference system
  - Simple trig, Plane coord system
- Geodetic Surveying: Larger areas, must account for earth's shape and dynamics
  - Curved reference, more complicated math



I. Earth Models

B. Geoid

Gravitational force = f(mass, distance)

Earth: non-homogeneous; mass anomalies

⇨ Lines of gravity are neither parallel nor straight.

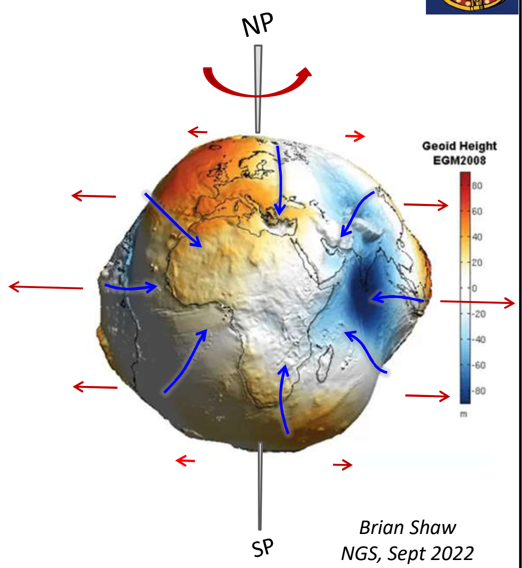
Centrifugal force = f(rotation, distance)

Minimum at Poles (dist=0),

Maximum at Equator (dist=RE)

Gravity = Gravitational+Centrifugal forces

Oblate spheroid: Flattened at poles, enlarged around Equator



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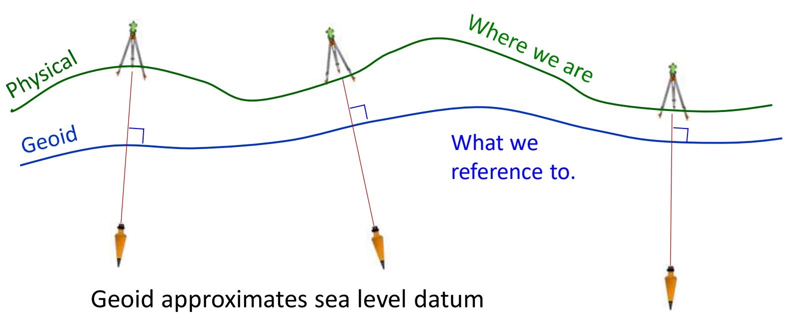
I. Earth Models

B. Geoid

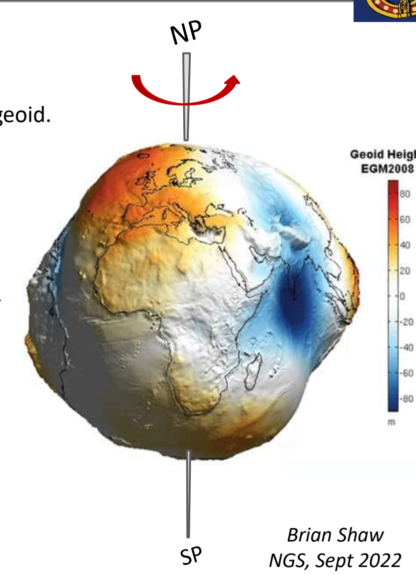
Gravity is perpendicular to the geoid

Centering a bubble or using a plumb bob orients equipment to the geoid.



Geoid approximates sea level datum

Connected to physical Earth by elevations - orthometric heights



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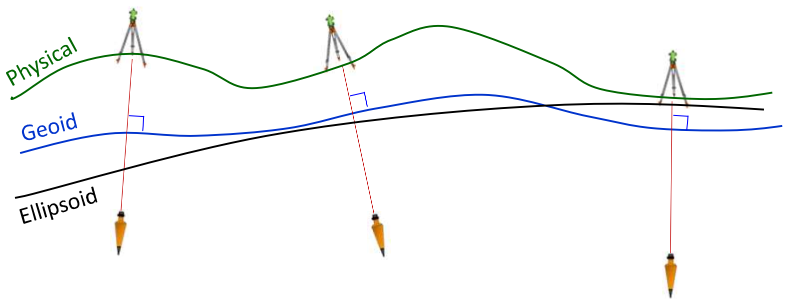
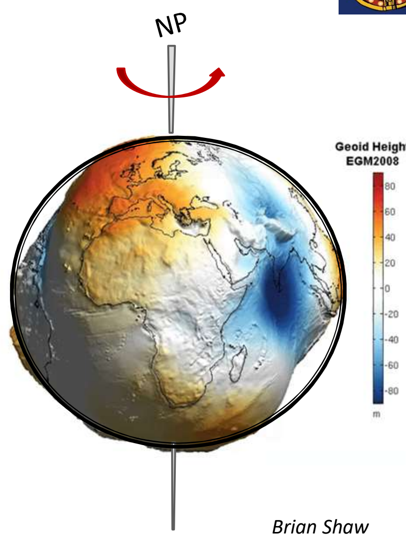
I. Earth Models

C. Ellipsoid

Mathematical 3D surface that is fit to geoid

Doesn't fit exactly; compromises





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I. Earth Models

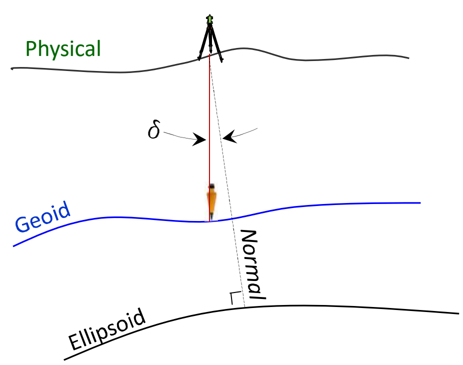
C. Ellipsoid

Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

Skewness - Deflection of the vertical, δ

Angle between directions of gravity and ellipsoid *normal*.

A normal is a line from the observer's position perpendicular to the ellipsoid



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I. Earth Models

C. Ellipsoid

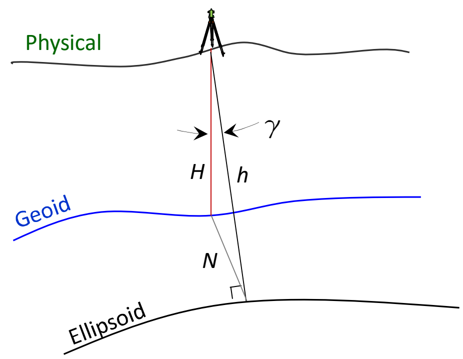
Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

Vertical separation

Heights between the surfaces

- H - Orthometric: geoid to ground
- N - Geoid: ellipsoid to geoid
- h - Ellipsoidal: ellipsoid to ground

h = H+N



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I. Earth Models

C. Ellipsoid

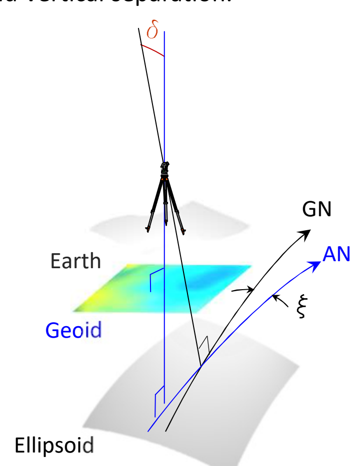
Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

LaPlace Corr'n, ξ

The component of δ that relates Geodetic N (GN) and Astronomic N (AN) at a particular latitude.

Was a common correction applied when performing astro obs for meridian determination.

See Sec 2-27 & -28, *2009 Manual*



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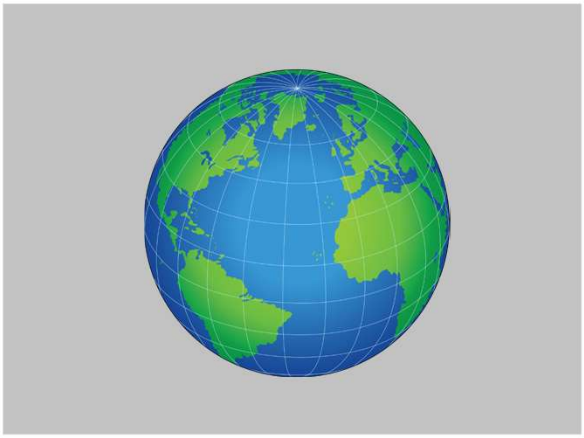


II. Spatial Systems

II. Spatial Systems

A. Three-Dimensional

1. Geodetic Coordinates

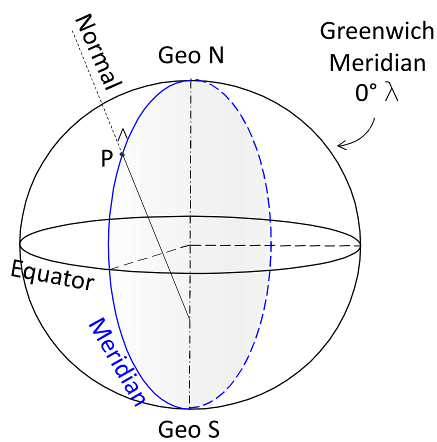
Reference defined by ellipsoid and fit.

NAD 83 - GRS 80 fit to Earth's mass center.

NAD 27 - Clarke 1866 fit to Meades Ranch, KS

Meridian — An elliptical section containing the normal and semi-minor axes. Defines Geodetic N at a point.

Geodetic meridians converge.



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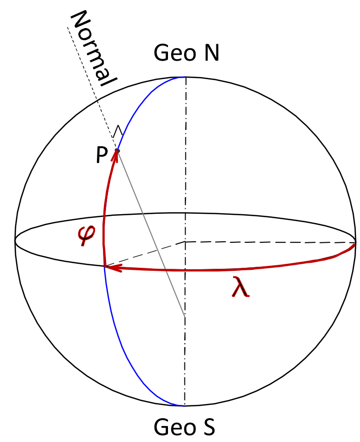
II. Spatial Systems

A. Three-Dimensional

1. Geodetic Coordinates

Longitude (λ) - Angle in Equatorial plane E or W from Greenwich Meridian
0°-180°W; 0°-180°E

Latitude (Φ) - Angle in meridian N or S of the Equator to the normal
0°-90°N; 0°-90°S



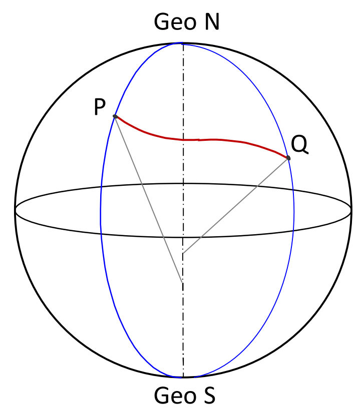
II. Spatial Systems

A. Three-Dimensional

1. Geodetic Coordinates

Disadvantages:

- Positions are expressed in angular values
- Distances are in angular values
- Elliptical geometry
  - Shortest distance between two points is a *geodesic* - shallow s-shape curve.



II. Spatial Systems

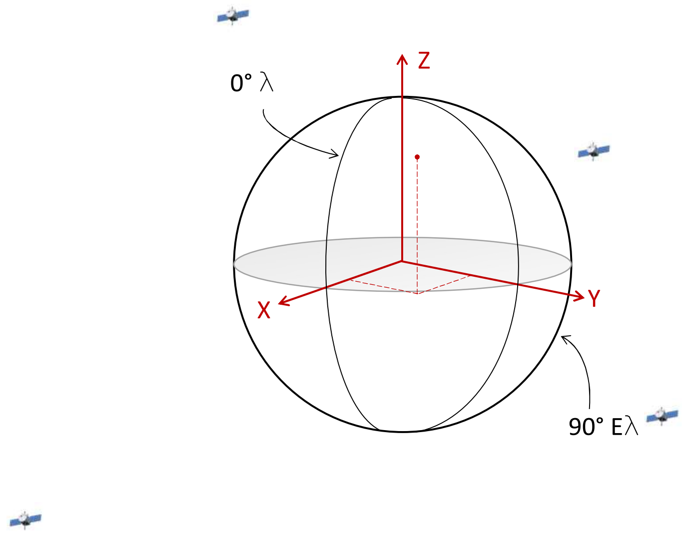
A. Three-Dimensional

2. Terrestrial Coordinate System - TCS

Three axis rectangular system

- Origin at Earth's mass center
- Coordinates are linear values

Used for satellite positioning



II. Spatial Systems

A. Three-Dimensional

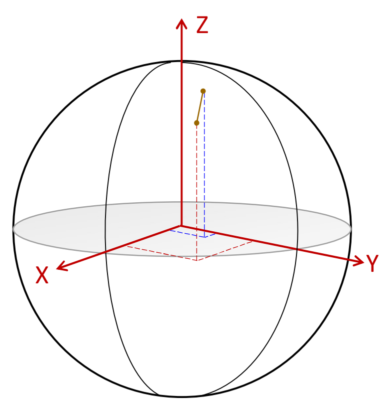
2. Terrestrial Coordinate System - TCS

Disadvantages:

- Huge coordinate values.
- Negative coordinates
- No "up" (vertical direction)
  - Top and bottom of vertical structures have different 3D coordinates.

Coordinates of control point *Ballaine* in Alaska:

X -2,295,158.170 meters
Y -1,441,664.364 meters
Z 5,754,524.764 meters



II. Spatial Systems

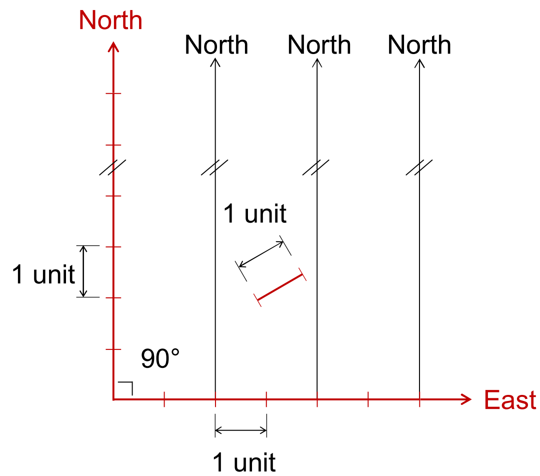
B. Two Dimensional

1. Plane

Characteristics

a. Orthogonal
b. Parallel north lines
c. Uniform scale in both directions

Comps are simple.



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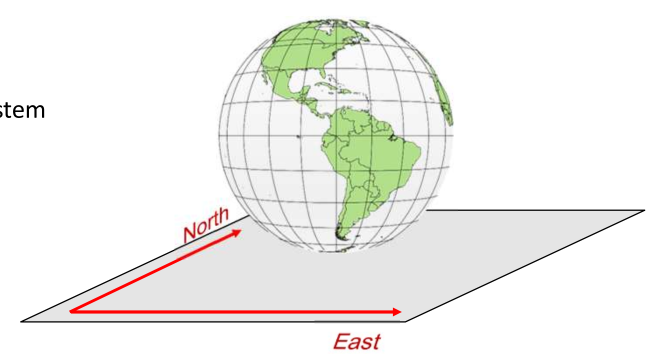
II. Spatial Systems

B. Two Dimensional

2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system



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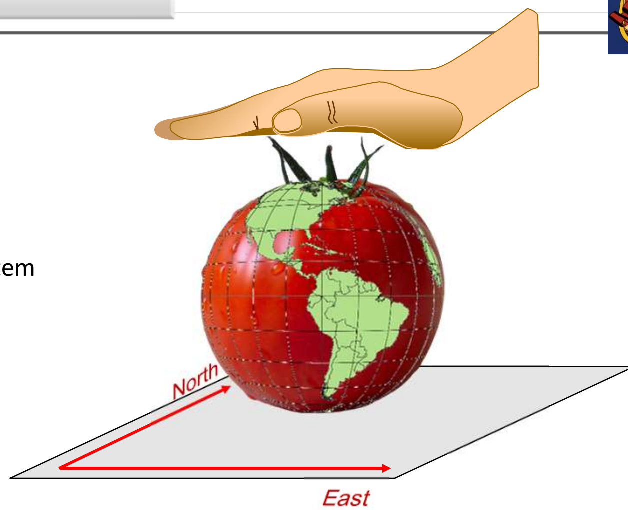
II. Spatial Systems

B. Two Dimensional

2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system



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II. Spatial Systems

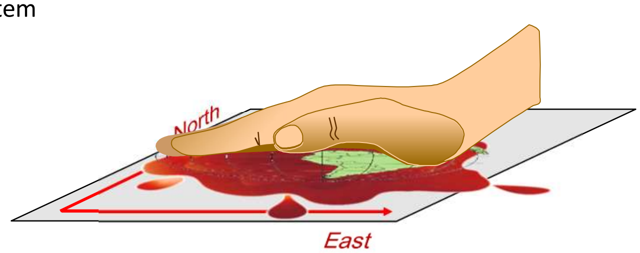
B. Two Dimensional

2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system

With a direct projection we get a distorted representation



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II. Spatial Systems

B. Two Dimensional

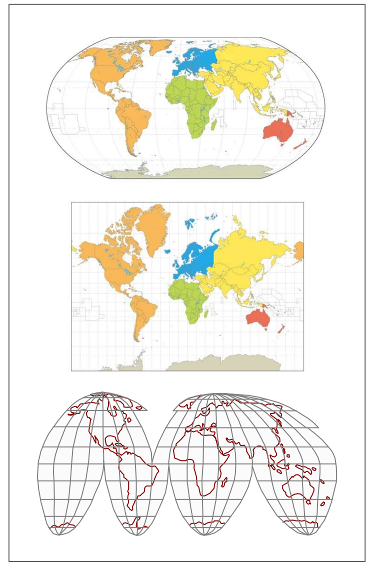
2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system

With a direct projection we get a distorted representation

Different mathematical projections distort different ways.



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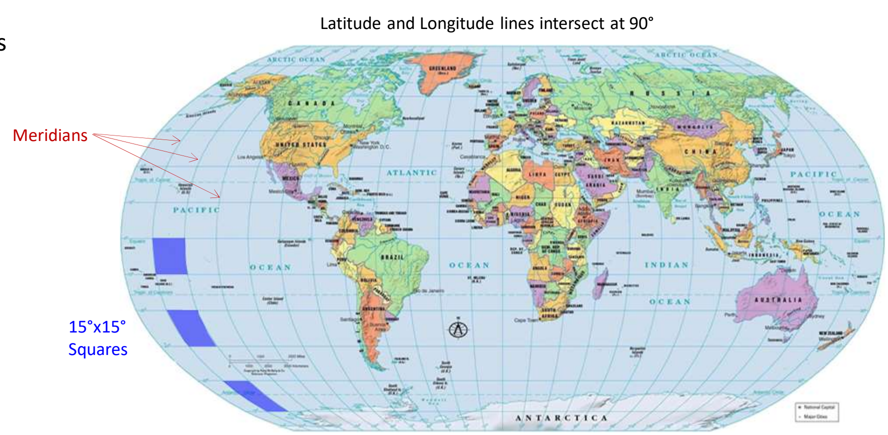
II. Spatial Systems

B. Two Dimensional

2. Distortions

Distances

Directions



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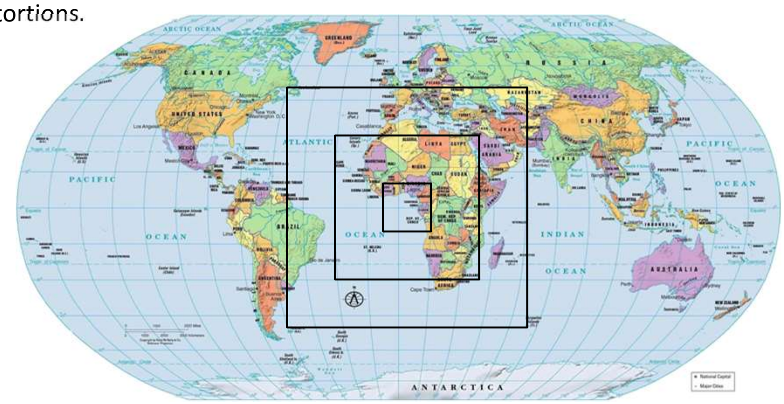
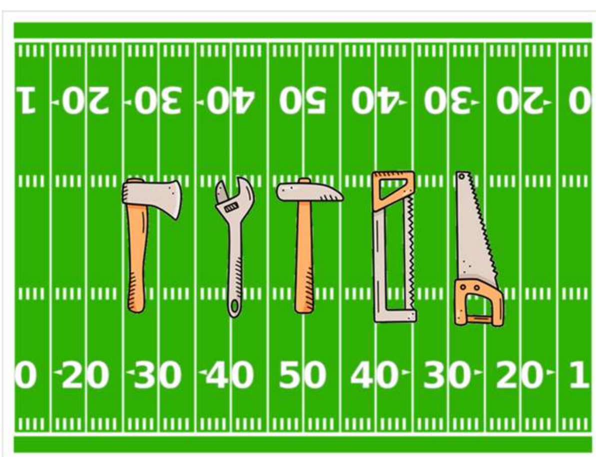
II. Spatial Systems

B. Two Dimensional

2. Distortions

The smaller the area projected, the smaller the distortions.





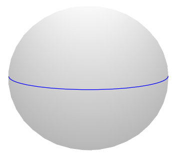
III. Grid Mechanics

III. Grid Mechanics

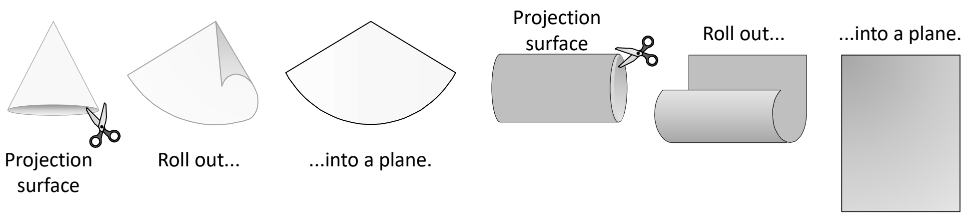
A. Projection Surfaces

To control or compensate distortions, we must project from a 3D mathematical surface to another mathematical surface that can be developed into a plane without introducing additional distortions.

The ellipsoid is the 3D surface.



A cone or cylinder are projection surfaces that can be rolled out flat into a 2D surface.



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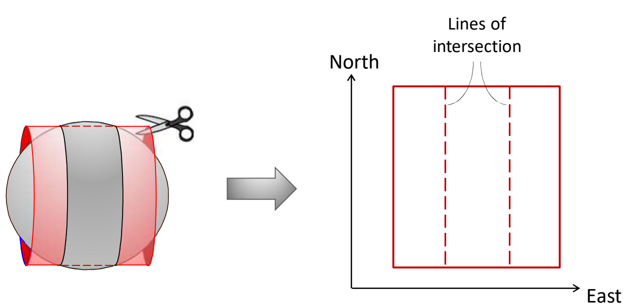
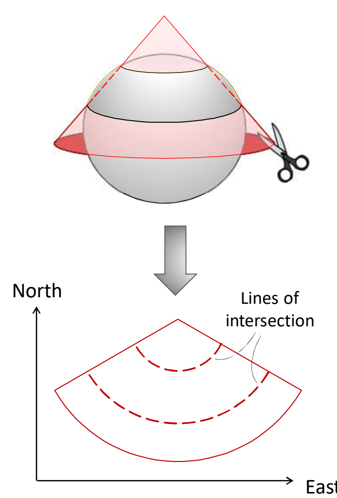
III. Grid Mechanics

A. Projection Surfaces

Secant projections are used

- Cone or cylinder cuts through the ellipsoid.
- Points are projected from the ellipsoid.
- Projection is rolled out
- Coordinate axes overlaid





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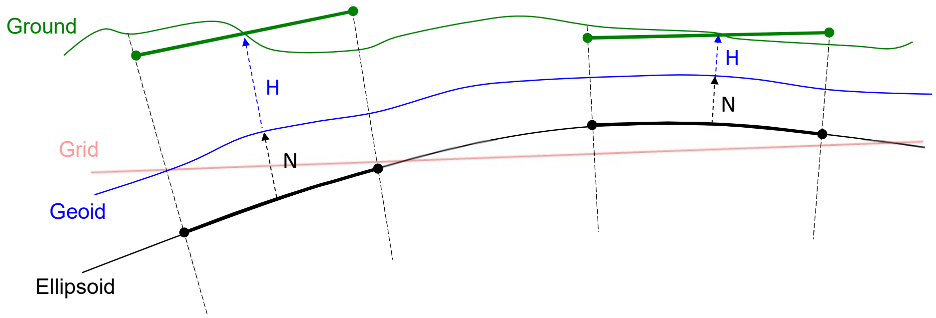
III. Grid Mechanics

B. Distance Distortion

Distances are reduced from Ground to Grid in two steps:

1. Horizontal ground to geodetic on the ellipsoid = f(orthometric, geoid heights)

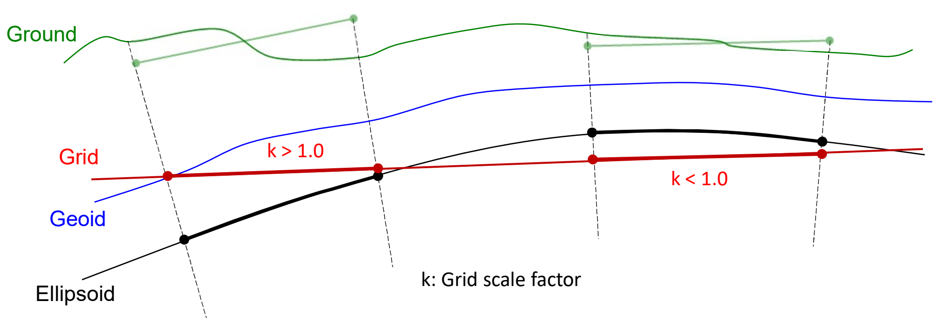


III. Grid Mechanics

B. Distance Distortion

Distances are reduced from Ground to Grid in two steps:

1. Horizontal ground to geodetic on the ellipsoid = f(orthometric, geoid heights)
2. Geodetic to grid = f(grid scale factor)

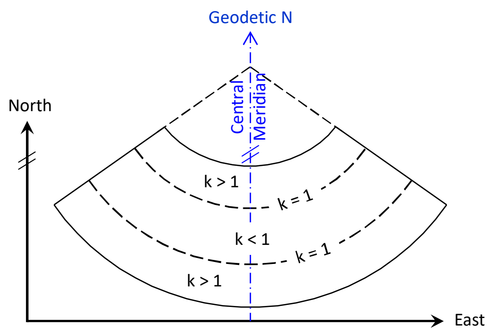


III. Grid Mechanics

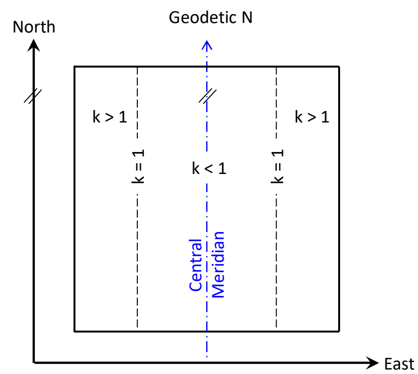
B. Distance Distortion

k: grid scale factor

Ellipsoid to grid



Conic: Scale varies N/S



Cylindric: Scale varies E/W

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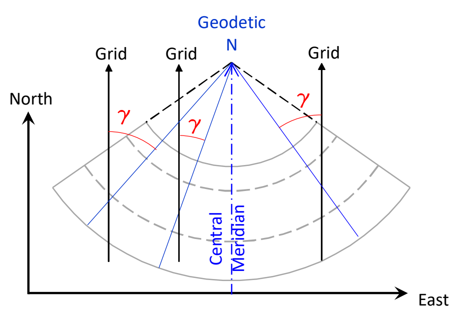
III. Grid Mechanics

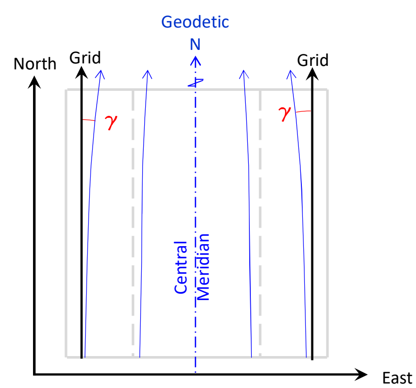
C. Direction Distortion

Convergence, γ, is angle between Grid and Geodetic North.

0° at CM, increases to E and to W; γ=f(Longitude)



Conic Projection



Cylindric Projection

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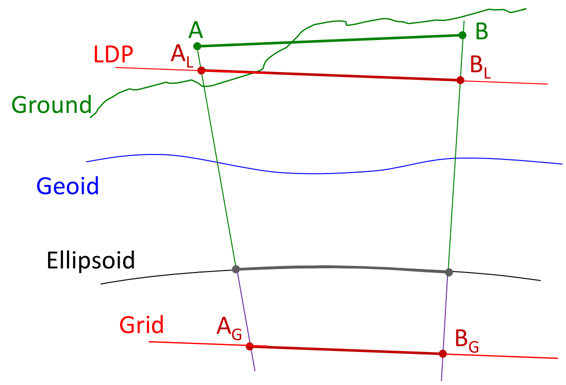
III. Grid Mechanics

D. Low Distortion Projection (LDP)

A low distortion projection covers a smaller area and brings the grid closer to Earth surface.

Distortions, ground to grid, are generally in the 1/40,000-1/60,000 range.



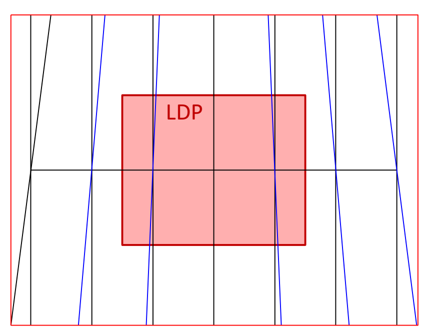
III. Grid Mechanics

D. Low Distortion Projection (LDP)

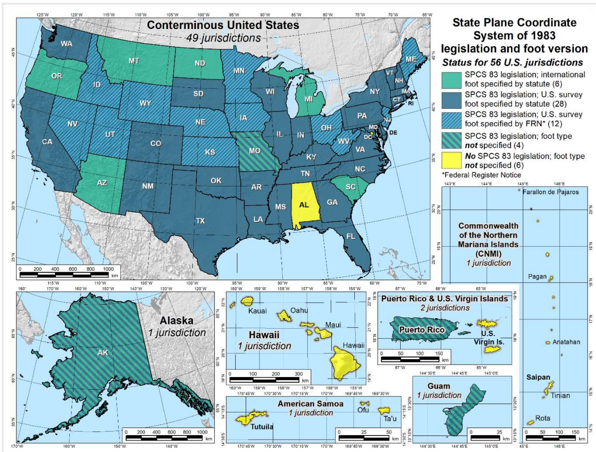
A low distortion projection covers a smaller area and brings the grid closer to Earth surface.

Because it covers less area E-W, convergence angles are smaller and more consistent.

Ground and Grid values, except for control purposes, can be treated as the same.



Smaller Convergence Angles

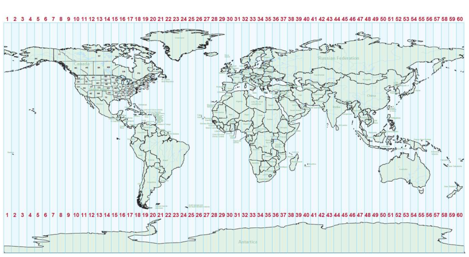


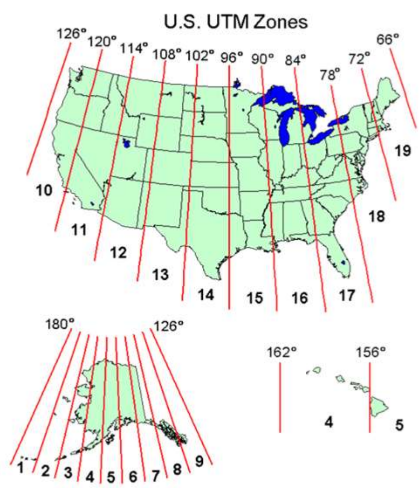
IV. Formal Coordinate Systems

IV. Formal Coordinate Systems

A. Universal Transverse Mercator (UTM)

- 60 adjacent cylindric projections around the Earth
- Each projection is 6° wide and runs from 80° S Lat to 84° N Lat.
- Numbered 1 to 60 from west to east and lettered C to X south to north
- Max distortion: 1/2500; k=0.9996 to 1.0004
- Same zones used in NAD 27 and NAD 83(xx).





Not NGS designed, but are included in NSRS and supported in *NCAT*.

IV. Formal Coordinate Systems

B. State Plane Coordinates (SPC)

Designed by NGS (C&GS), included in NSRS and supported in NCAT

1. NAD 27

   Development began in 1930s

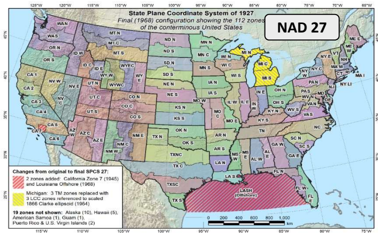
   Maximum distance distortion 1/10,000 (ellipsoid to grid)

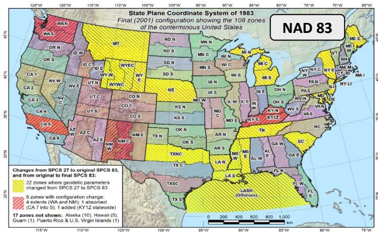
   k = 0.9999 to 1.0001

   →multiple zones in most states

2. NAD 83(xx)

   Some zone reshuffling





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IV. Formal Coordinate Systems

C. Low Distortion Projections (LDP)

Many states currently have NAD83(xx) based LDP systems.

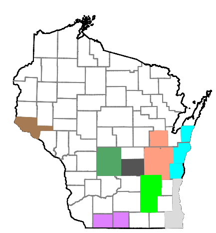
By counties or regions

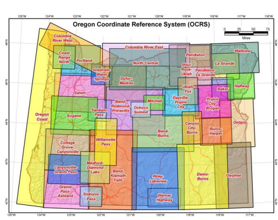
Locally designed, not NGS supported.

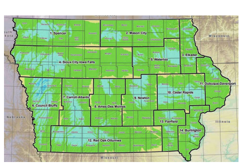
Most states will have LDPs for the new 2022 datums.

NGS designed and supported.

Will be in NGS software (eg, *NCAT*)







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V. Ground and Grid

What's the beef?

V. Ground and Grid

A. Distortion Compensation

1. Distance

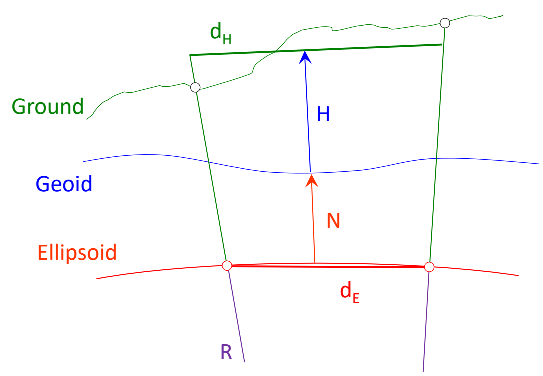
Two steps

a. Ground to ellipsoid

$$EF = \frac{R}{R + H + N}$$

$$d_E = d_H \times EF$$

- d_H Horizontal ground distance
- d_E Ellipsoidal (geodetic) distance
- EF Elevation Factor
- R Mean earth radius
- H Orthometric ht (elev)
- N Geoid height
- k Scale factor



R = 20,902,000 ft = 6,371,000 m (approx.)

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V. Ground and Grid

A. Distortion Compensation

1. Distance

Two steps

b. Ellipsoidal to grid

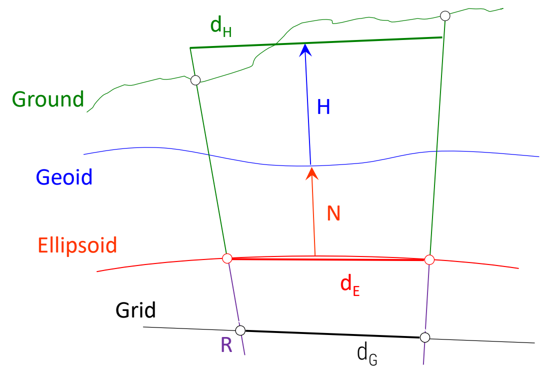
$$d_G = d_E \times k$$

d_G Grid distance
d_E Ellipsoidal (geodetic) distance
k Grid scale factor

c. Combined factor

$$CF = EF \times k$$

$$d_G = d_H \times CF$$



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V. Ground and Grid

A. Distortion Compensation

2. Direction

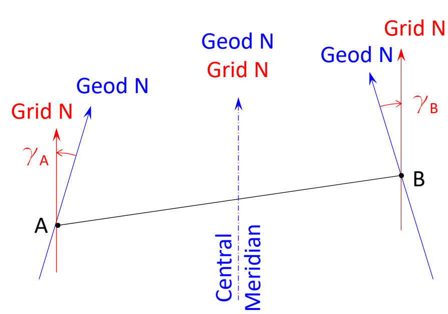
The convergence angle, γ, is **from** Geodetic N **to** Grid N

It is positive (cw) East of the CM, negative (ccw) West of the CM

To convert Geodetic (Ground) direction to Grid:

$$t = \alpha - \gamma$$

t Grid azimuth
α Geodetic azimuth
γ Convergence



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V. Ground and Grid

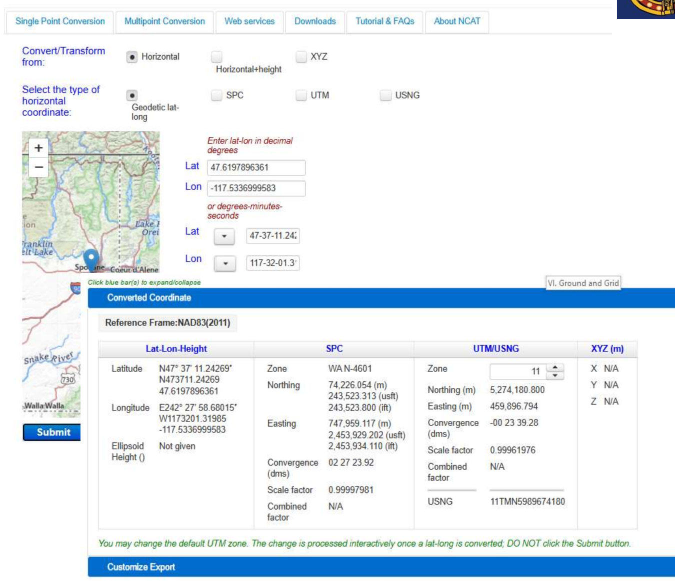
B. Reduction Elements

Where do we get the ortho and geoid heights, scale, and convergence angles?

1. NGS software (*Geodetic Tool Kit*):
   - *NCAT*[1]
   - *GEOIDXX*

Ortho heights from USGS topoquads

[1]*NCAT* does not currently support local LDPs. When NATRF2022 is adopted, *NCAT* will include NGS-accepted LDPs.



V. Ground and Grid

B. Reduction Elements

2. NSRS Datasheet

```
DH4844  DESIGNATION -  BALLAINE
DH4844  PID         -  DH4844
DH4844  STATE/COUNTY-  AK/FAIRBANKS NORTH STAR
DH4844  COUNTRY     -  US
DH4844  USGS QUAD   -  FAIRBANKS D-2 NW (2013)
DH4844
DH4844                        *CURRENT SURVEY CONTROL
DH4844
DH4844* NAD 83(2011) POSITION- 64 55 39.44098(N) 147 51 56.58599(W)   ADJUSTED
DH4844* NAD 83(2011) ELLIP HT-    253.779 (meters)        (06/27/12)   ADJUSTED
DH4844* NAD 83(2011) EPOCH    -   2010.00
DH4844* NAVD 88 ORTHO HEIGHT -    244.4    (meters)      802.     (feet) GPS OBS
DH4844
DH4844  NAVD 88 orthometric height was determined with geoid model    GEOID99
DH4844  GEOID HEIGHT    -            11.162 (meters)                   GEOID99
DH4844  GEOID HEIGHT    -             9.481 (meters)                   GEOID12B
DH4844  NAD 83(2011) X  - -2,295,158.170 (meters)                      COMP
DH4844  NAD 83(2011) Y  - -1,441,664.364 (meters)                      COMP
DH4844  NAD 83(2011) Z  -  5,754,524.764 (meters)                      COMP
DH4844  LAPLACE CORR    -           -12.35  (seconds)                  DEFLEC12B
DH4844
```

V. Ground and Grid

B. Reduction Elements

2. NSRS Datasheet

```
DH4844  Network accuracy estimates per FGDC Geospatial Positioning Accuracy
DH4844  Standards:
DH4844         FGDC (95% conf, cm)     Standard deviation (cm)     CorrNE
DH4844            Horiz  Ellip           SD_N   SD_E   SD_h      (unitless)
DH4844  -------------------------------------------------------------------
DH4844  NETWORK    4.03   6.51           1.76   1.52   3.32      -0.04506030
DH4844  -------------------------------------------------------------------

DH4844. The following values were computed from the NAD 83(2011) position.
DH4844
DH4844;                     North         East     Units Scale Factor Converg.
DH4844;SPC AK 3      - 1,218,533.229   411,764.487   MT  0.99999529   -1 41 24.1
DH4844;UTM  06       - 7,200,668.407   459,066.185   MT  0.99962051   -0 47 03.0
DH4844
DH4844!              -  Elev Factor  x  Scale Factor =  Combined Factor
DH4844!SPC AK 3      -   0.99996030  x   0.99999529  =   0.99995559
DH4844!UTM  06       -   0.99996030  x   0.99962051  =   0.99958082
DH4844
```

At *Ballaine's* elevation

V. Ground and Grid

C. Variations

1. Elevation factor, EF

$$EF = \frac{R}{R + H + N}$$

N doesn't change much so can generally use a single value over the project area.

Depending on terrain, H can be:

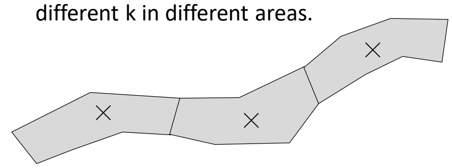
project area average – use for all lines

computed average for each line

2. Grid scale, k

For relatively small projects, a single value at project center could be used.

Larger/longer projects would require applying different k in different areas.



SPC/UTM - use approx. lat & long with *NCAT* to determine k.

V. Ground and Grid

D. SPC Example

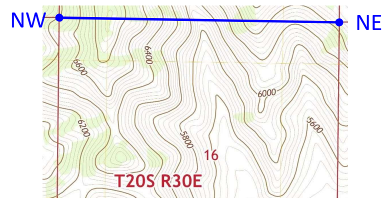
North line Sec 16 T20S R30E Gila & Salt River PM

Measurements

Distance: 5253.64 ft

Bearing: S88°52’18”E

Determine AZ E Zone SPC grid distance & direction.



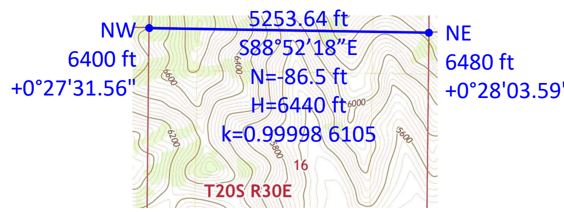
From USGS *Swede Peak* topoquad:

Corner	NW	NE
Lat	31°41'53"	31°41'51"
Long	-109°17'37"	-109°16'36"
Elev (ft)	6400	6480

V. Ground and Grid

D. SPC Example

Corner	NW	NE
From *NCAT*		
Conv	+0°27'31.56"	+0°28'03.59"
k	0.99998 445	0.99998 776
From *GEOID18*		
N (m)	-26.366	-26.388



$$N = \frac{-26.366\ m + (-26.388\ m)}{2} = -26.377\ m$$

$$-26.377\ m \times \frac{39.37\ in}{1\ m} \times \frac{1\ ft}{12\ in} = -86.5\ ft$$

$$H = \frac{6400\ ft + 6480\ ft}{2} = 6440\ ft$$

$$k = \frac{0.99998\ 445 + 0.99998\ 776}{2}$$
$$= 0.99998\ 6105$$

V. Ground and Grid

D. SPC Example

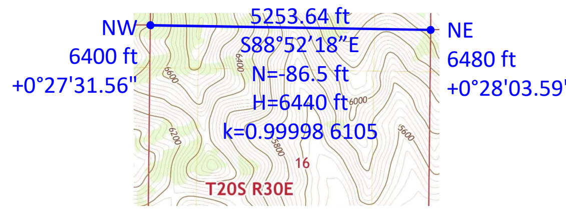
1. Distance

   a. Ground to ellipsoid

$$EF = \frac{20,902,000\ ft}{20,902,000\ ft + 6440\ ft + (-86.5\ ft)}$$

$$= 0.99969\ 61262$$

$$d_E = 5253.64\ ft \times 0.99969\ 61262 = 5252.0436\ ft$$



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V. Ground and Grid

D. SPC Example

1. Distance

   a. Ground to ellipsoid

$$EF = \frac{20,902,000\ ft}{20,902,000\ ft + 6440\ ft + (-86.5\ ft)}$$

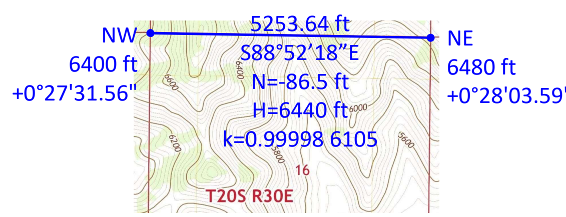
$$= 0.99969\ 61262$$

$$d_E = 5253.64\ ft \times 0.99969\ 61262 = 5252.0436\ ft$$

   b. Ellipsoid to grid

$$d_G = 5252.0436\ ft \times 0.99998\ 6105$$

$$= \underline{5251.971\ ft}$$



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V. Ground and Grid

D. SPC Example

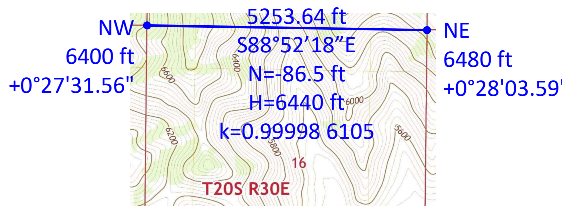
1. Distance

a. Ground to ellipsoid

$$EF = \frac{20,902,000\ ft}{20,902,000\ ft + 6440\ ft + (-86.5\ ft)}$$

$$= 0.99969\ 61262$$

$$d_E = 5253.64\ ft \times 0.99969\ 61262 = 5252.0436\ ft$$



b. Ellipsoid to grid

$$d_G = 5252.0436\ ft \times 0.99998\ 6105$$
$$= \underline{5251.971\ ft}$$

Ground to grid distortion

$$\frac{5253.64 - 5251.971}{5251.971} = \frac{1.669}{5251.971} = \frac{1}{3150}$$

V. Ground and Grid

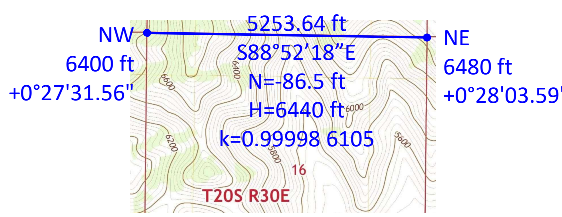
D. SPC Example

2. Direction

NW to NE

Convert bearing to azimuth

$$Az = 180°00'00'' - 88°52'18''$$
$$= 91°07'42''$$

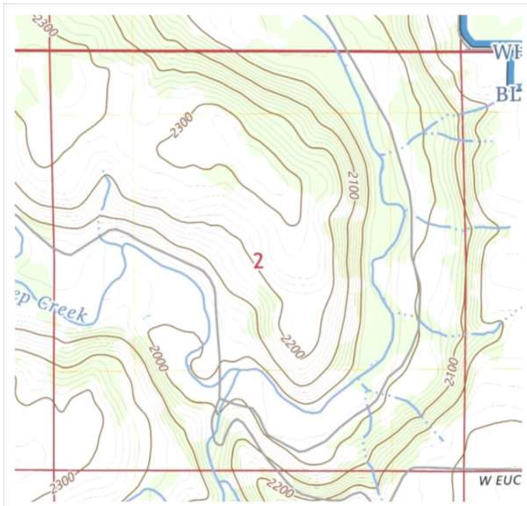


Convert to Grid Az

$$Grid\ Az = 91°07'42'' - (+0°27'31.6'')$$
$$= 90°40'10.4''$$

Convert to bearing

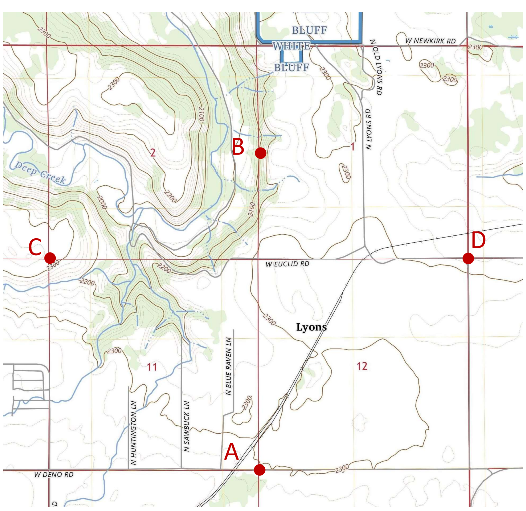
$$\beta = 180°00'00'' - 90°40'10.4''$$
$$= 89°19'49.6''$$
$$Grid\ brng = S89°19'50''E$$



VI. Grids and PLSS Lost Corners

VI. Grids and PLSS Lost Corners

A. Cardinal Equivalents



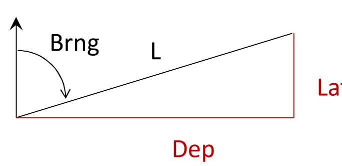
Proportionate Measurement

Recreating original locations based on record and contemporary measurements.

Proportioning.

PLSS Manual states that proportioning must be done in *cardinal directions*

True N-S (Lat) & True E-W (Dep)



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VI. Grids and PLSS Lost Corners

B. True Direction Reference

If working in a grid system, must convert Grid directions to True directions.

Grid direction is constant along the line

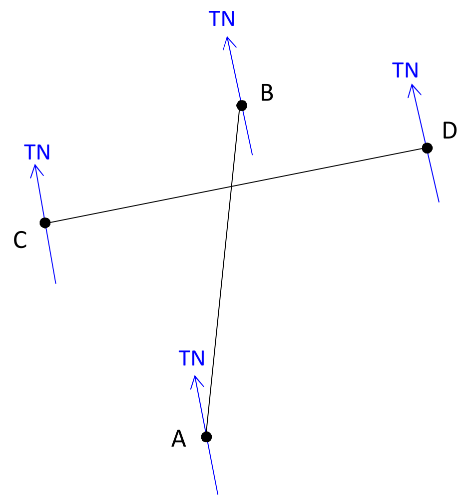
True direction is not because True meridians converge

That means:

Card Equiv AB ≠ Card Equiv BA

and

Card Equiv CD ≠ Card Equiv DC



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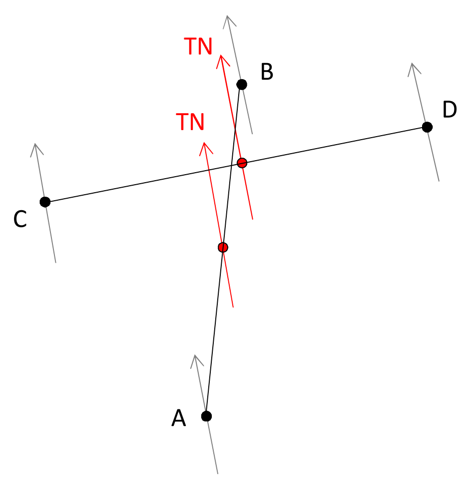
VI. Grids and PLSS Lost Corners

B. True Direction Reference

2009 Manual

2-11. The basis for reporting direction is called true mean bearing. Stated in terms of angular measure referred to true meridian north, it is referenced to the true meridian at the point of record. In practical application of the concept, the point of record for determining the bearing of a line can be said to be the meridian at the midpoint of the line of sight between the end points.

Use midpoint bearings



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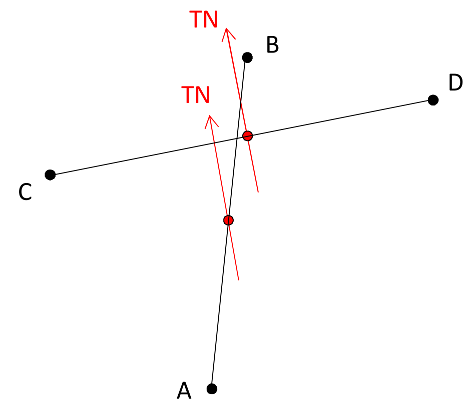
VI. Grids and PLSS Lost Corners

B. True Direction Reference

Two ways to compensate for convergence

Method 1. Before proportioning

Method 2. After proportioning



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VI. Grids and PLSS Lost Corners

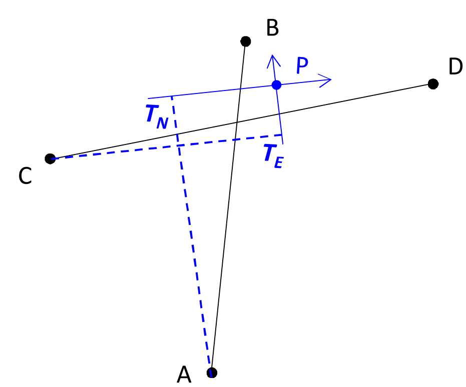
B. True Direction Reference

Method 1. Convert Before Proportioning

For each line

- Compute Grid Bearing from coordinates
- Using average coords for midpoint, determine its convergence
- Convert Grid to True Bearing
- Compute cardinal equivalents

Proceed with proportioning



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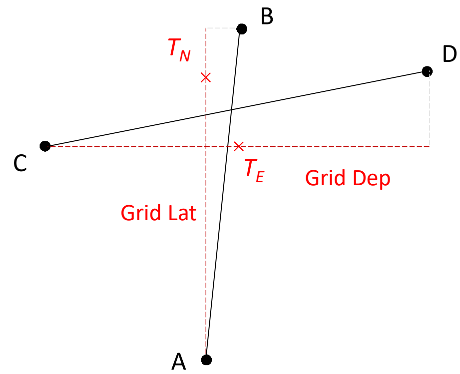
VI. Grids and PLSS Lost Corners

B. True Direction Reference

Method 2. Convert After Proportioning

- For each line
  - Compute Grid Bearing from coordinates
  - Using average coords for midpoint, determine its convergence
  - Compute Grid equivalents
  - Compute proportionate distances and set temporary points.



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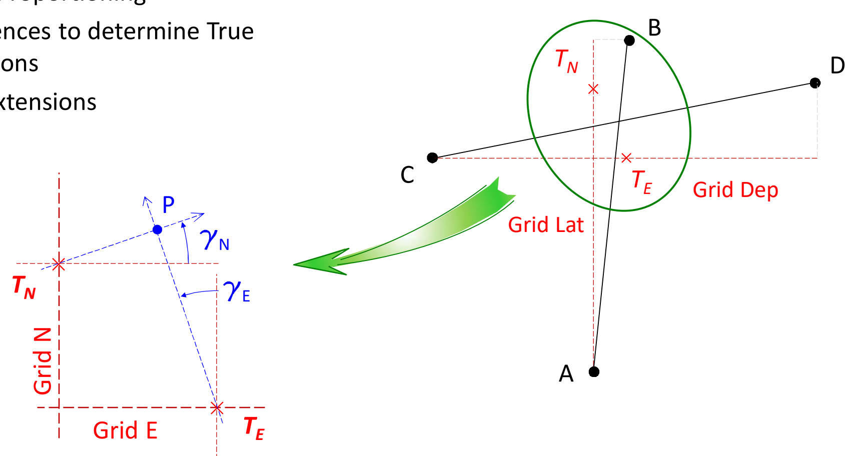
VI. Grids and PLSS Lost Corners

B. True Direction Reference

Method 2. Convert After Proportioning

- Use midpoint convergences to determine True E-W and N-S extensions
- Extend and intersect extensions



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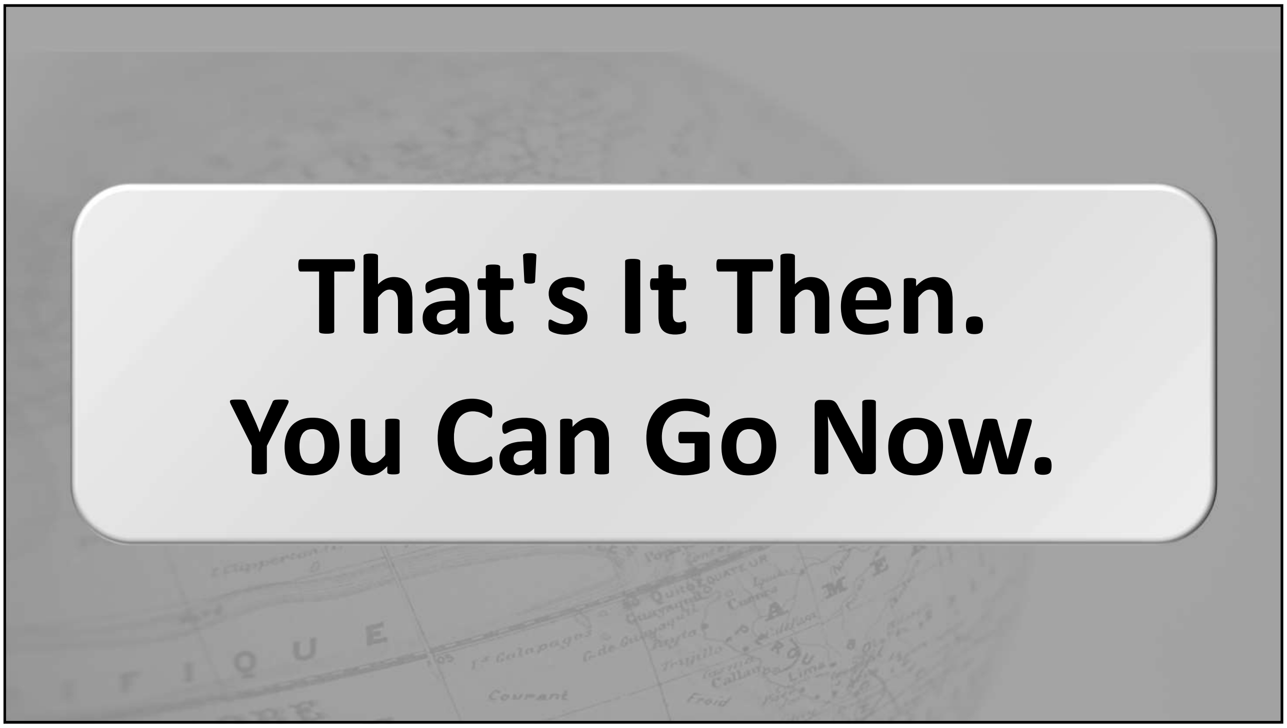
I. Earth Models
II. Spatial Systems
III. Grid Mechanics
IV. Formal Coordinate Systems
V. Ground and Grid
VI. Grids and PLSS Lost Corners

Grid and Ground
Simple, Right?


That's It Then.
You Can Go Now.