WESTERN REGIONAL SURVEY CONFERENCE

MARCH 18 - 21, 2026

Errors: Measurement to Product

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Semi-Retired and a burden on my wife

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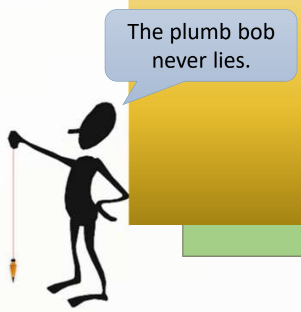


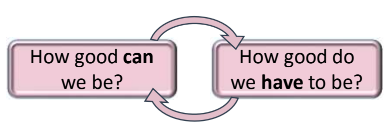
Regardless the equipment used, the personnel involved, survey measurements will include errors.

Almost every survey operation at some point will require measurements. That means the operation results will, in part, depend on the measurement errors.

How much error is acceptable? It depends.

Depending on the operation, the error level is either up to the surveyor or dictated by some rule or law.

Either way requires understanding where errors come from, how they behave, how they can be controlled, and determining their effect on our final decisions.



WESTERN REGIONAL SURVEY CONFERENCE
MARCH 18 - 21, 2026

Subjects

I. Measurement Errors
II. Random Errors
III. Least Squares
IV. Standards and Specifications

I. Measurement Errors

A. Quality
B. Error Tenets
C. Sources
D. Types
E. Minimizing

I. Measurement Errors

A. Quality

Accuracy

Absolute nearness to true value; measurement with minimal errors.

Precision

Repeatability; spread of a measurement set.

Similar error behavior in repeated measurements

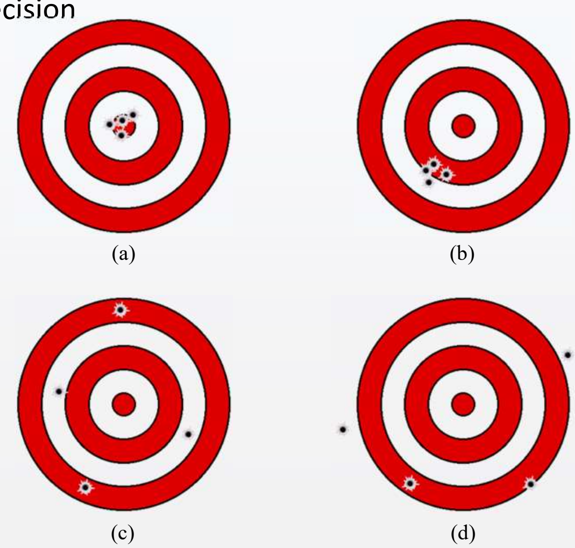
Resolution

The smallest division to which the measurement can be expressed based on the instrumentation used.

I. Measurement Errors

A. Quality

Accuracy and Precision



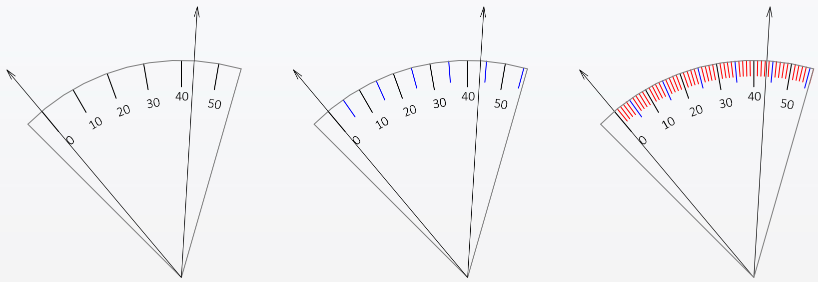
(a) (b) (c) (d)

I. Measurement Errors

A. Quality

Resolution

Related to *degree* of accuracy



direct: 10°
interpolate: 1°

direct: 5°
interpolate: 1°

direct: 1°
interpolate: 0.5°

I. Measurement Errors

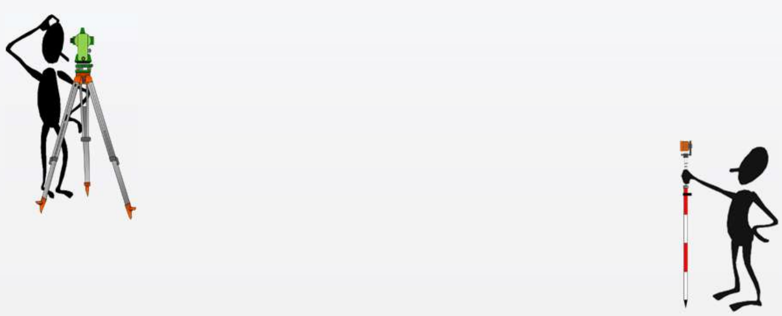
B. Error Tenets

No measurement is exact.

Every measurement contains errors.

The true value of a measurement is never known.

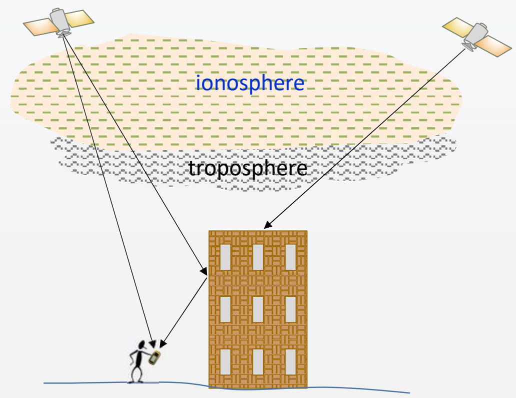
The exact error present in a measurement is unknown.



I. Measurement Errors

C. Error Sources - Where they come from

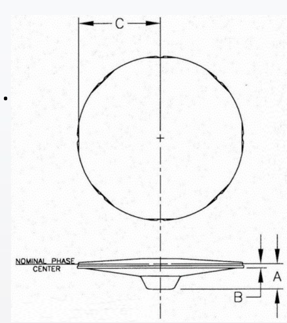
1. Natural
   Environment within which a measurement is made

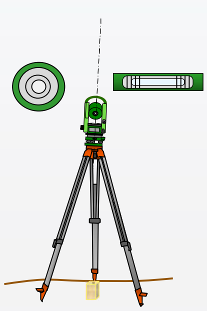


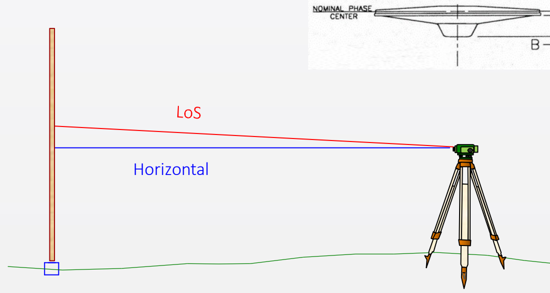
I. Measurement Errors

C. Error Sources - Where they come from

2. Instrumental
   Measuring equipment condition; calibration.





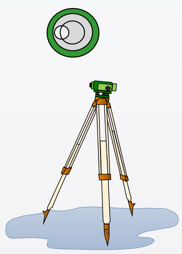


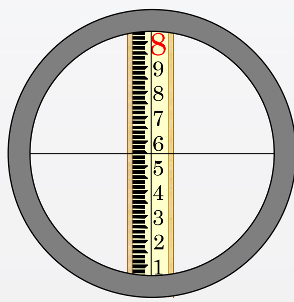
I. Measurement Errors

C. Error Sources - Where they come from

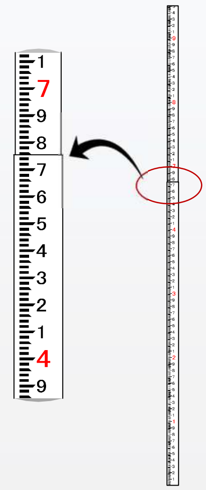
3. Personal

    People making the measurement





Record 8.55.
Common newbie mistake.



I. Measurement Errors

D. Error Types - How they behave

1. Mistake

    Carelessness or misunderstanding

2. Systematic

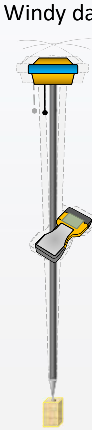
    Conform to mathematical or physical law

3. Random

    Remaining error after mistakes & systematic errors are eliminated

    Tend to be small; as likely positive as negative.

Windy day

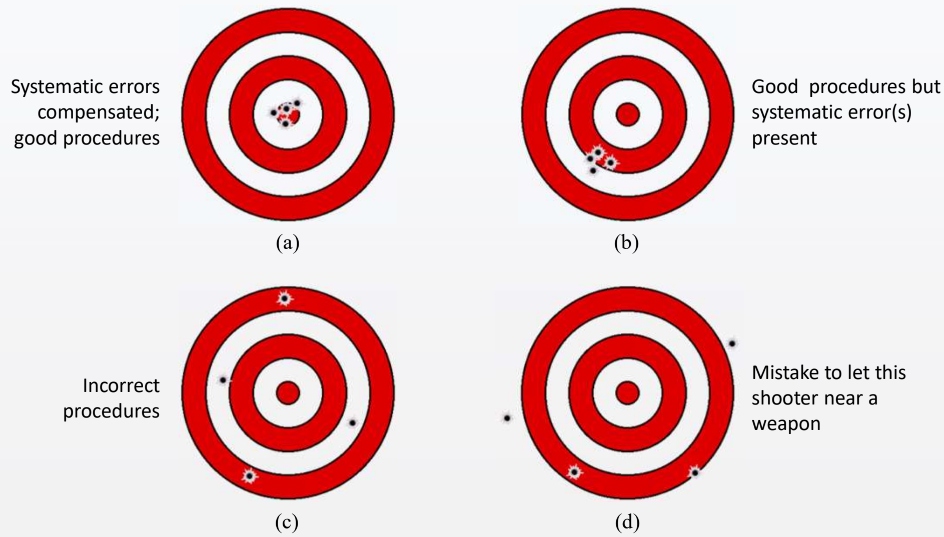


Entered wrong height

Set up on wrong point

I. Measurement Errors

D. Error Types - How they behave



I. Measurement Errors

E. Minimizing

1. Mistake

Generally, with a single measurement, can't tell if a mistake was made.

Remeasure until all measurements are within an acceptable tolerance

Tolerance can be defined as:

- A specific spread, eg, angles must all be within 10"
- A precision level, eg, distances must be within 1/5000
- Different based on equipment or personnel, eg, angles within 10" for experienced operator, 25" for new tech.

Reject measurements outside the tolerance to avoid biasing good measurements.

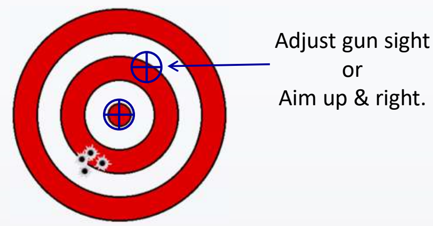
I. Measurement Errors

E. Minimizing

2. Systematic

Apply a correction:

- Mathematic
- Mechanical
- Procedure



Specific surveying procedures are used to:

- ensure measurement is made correctly
- allow some systematic errors to cancel

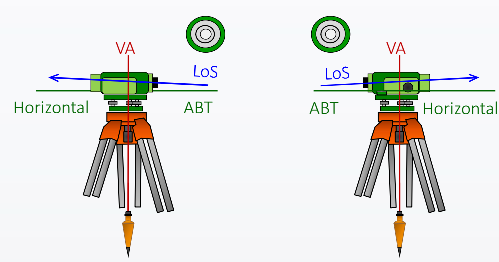
I. Measurement Errors

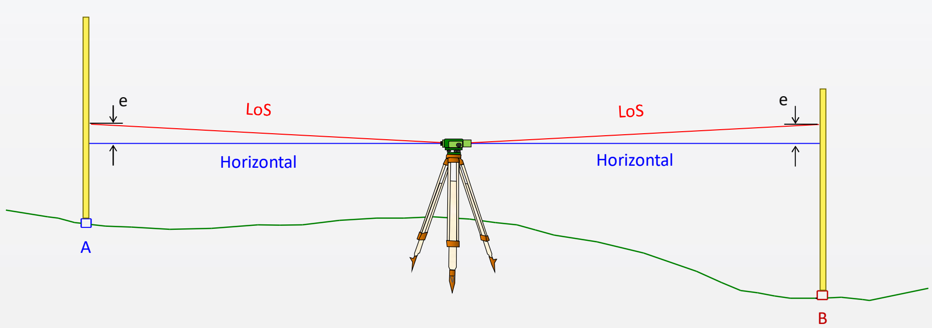
E. Minimizing

2. Systematic

Example: Level collimation error

- Balance BS and FS distances
- Two-peg test
  - Adjust
  - Math corr'n





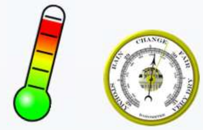
I. Measurement Errors

E. Minimizing

2. Systematic

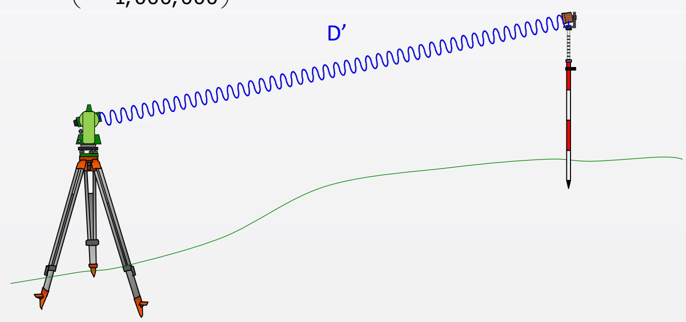
Example: Atmospheric conditions electronic distance

Math correction, ppm



$$ppm = 278.96 - \frac{10.5 \times P}{1 + 0.002175 \times T}$$

$$D = D' \times \left(1 + \frac{ppm}{1,000,000}\right)$$



I. Measurement Errors

E. Minimizing

3. Random

Appropriate equipment

Knowledgeable personnel

Favorable conditions

Repeat measurements

How many times?

Example: FGCS Triangulation angle measurement specs

Order/Class	1st	2nd / I	2nd / II	3rd / I	3rd / II
Num Positions	16	16	8 or 12*	4	2
Max Std Dev of Mean	0.4”	0.5”	0.8”	1.2”	2.0”
Reject Limit from Mean	4”	4”	5”	5”	5”

a position is 1 D/R measurement

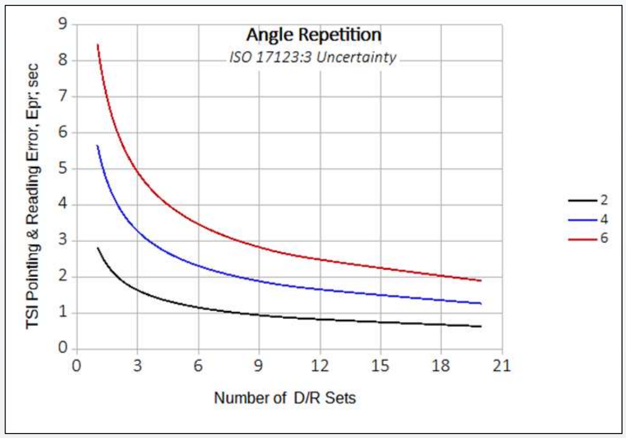
I. Measurement Errors

E. Minimizing

3. Random

Comparison of different resolution Total Stations

Pointing and reading error



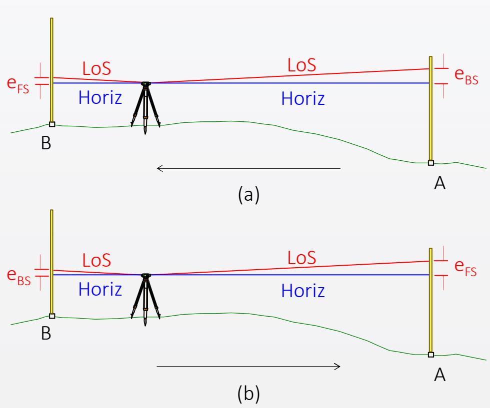
I. Measurement Errors

E. Minimizing

3. Random

A measurement must be independent.

Not changing conditions allows mistakes and systematic errors to be repeated.



Leveling

From a single set up:

(a) BS A and FS B

(b) BS B and FS A

Collimation error does not compensate.

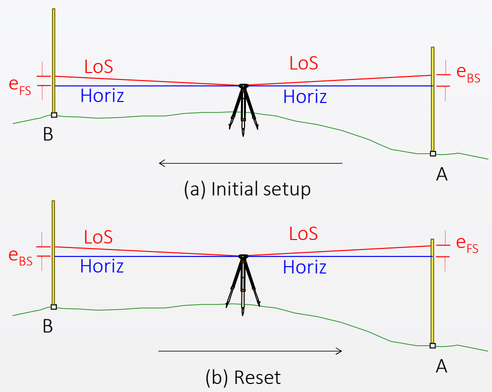
I. Measurement Errors

E. Minimizing

3. Random

A measurement must be independent.

Change conditions (reset inst, resight tagets, diff day, combo, etc).



Leveling

As a minimum, reset level between runs:

(a) Balance BS & FS dists;
BS A then FS B

(b) Reset with BS/FS dists balanced;
BS B then FS A

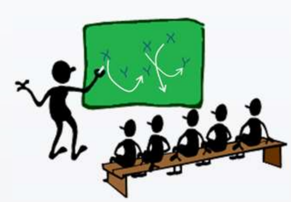
I. Measurement Errors

E. Minimizing

3. Random

To minimize errors:

- Favorable conditions
- Use proper equipment
- Trained personnel
- Correct procedures
- Repeat measurements



This will minimize but not eliminate errors.

Only random errors will be left and they should be small.

II. Random Errors

A. Basic Analysis
B. Error Propagation
C. Summary

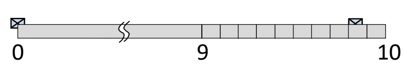
II. Random Errors

A. Basic Analysis

1. Terms

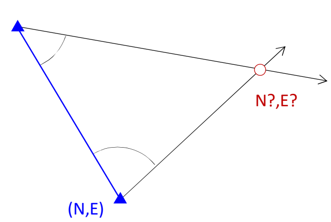
Direct measurement

Measure the unknown with instrumentation



Indirect measurement

Determine the unknown from measurements and computations



Want coordinates, but can't measure them directly.
Perform an angle-angle intersection.

II. Random Errors

A. Basic Analysis

1. Terms

Redundancy

Also known as a degree of freedom (df)

A measurement beyond what is needed to determine a quantity.

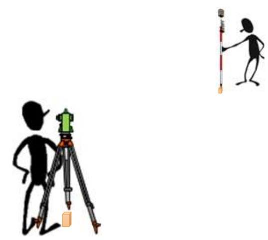
Example: a horizontal distance is measured once

- 1 measurement
- 1 unknown – the distance
- 0 redundancy

Measure again

- 2 measurements
- 1 unknown
- 1 redundancy

etc.



II. Random Errors

A. Basic Analysis

1. Terms

Discrepancy

Difference between any two measurements in a set

Most Probable Value, MPV

The most likely value of the unknown based on the measurement set.

If all measurements are equal quality, then the MPV is the arithmetic average.

$$M = \frac{\sum m_i}{n}$$

m: measurement

n: number of measurements

II. Random Errors

A. Basic Analysis

1. Terms

Residual, v

The difference between the MPV and a measurement.

$$v_i = M - m_i$$

Least Squares

The MPV is the value which minimizes the sum of the squared residuals.

$$\sum(v_i^2) = \min$$

Least sum of the *Squares*.

Any other number will increase the sum.

This is based on statistical behavior of random errors.

II. Random Errors

A. Basic Analysis

1. Terms

Standard Deviation, σ

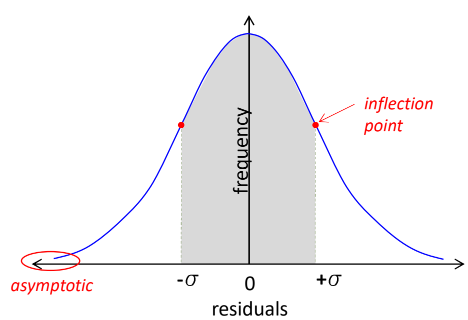
The *Normal Distribution* (aka, *Bell*) curve is a graph of residuals frequency.

Area under the curve is the total measurement probability

σ is approx. 68% of the area under the *Normal Distribution* curve.

Precision indicator of a measurement set.

$$\sigma = \sqrt{\frac{\sum(v_i^2)}{(n-1)}}$$



II. Random Errors

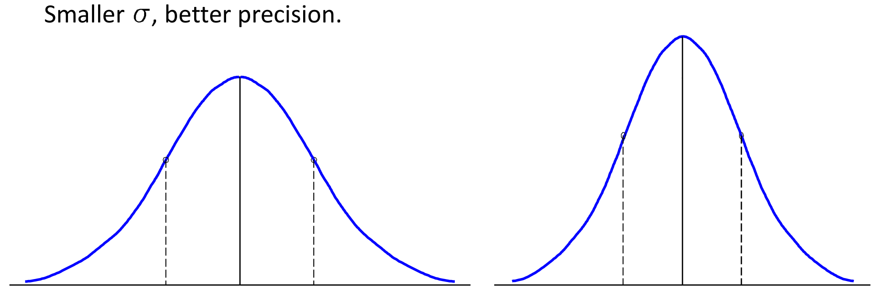
A. Basic Analysis

1. Terms

Standard Deviation, σ

The smaller the σ, the less dispersed the measurements.

Smaller σ, better precision.



Measurement set 1
Larger σ

Measurement set 2
Smaller σ, better precision

II. Random Errors

A. Basic Analysis

1. Terms

Confidence Interval, CI

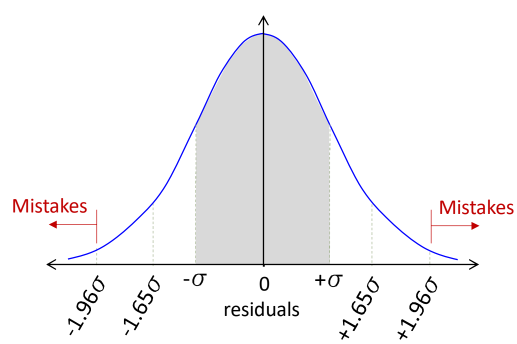
Certainty that a measurement will fall within a range.

$\pm\sigma$ is a 68% CI

Other common CI are:

CI	*Value*	
90%	≈ 1.65(σ)	
95%	≈ 1.96(σ)	aka *2 sigma**
100%?		

*95% CI is often used to find mistakes.



II. Random Errors

A. Basic Analysis

1. Terms

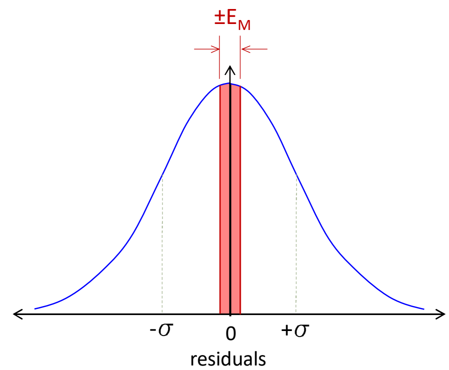
Standard Error of the Mean, E_M

σ is for the measurement set

E_M is the expected error in the MPV

An expected accuracy indicator.

$$E_M = \frac{\sigma}{\sqrt{n}}$$



II. Random Errors

A. Basic Analysis

2. Example

Measurement		v = M-m_i	v^2
num	m		
1	45.66		
2	45.68		
3	45.66		
4	45.65		
sums:	182.65		

$$df = 4 - 1 = 3$$

II. Random Errors

A. Basic Analysis

2. Example

Measurement			
num	m	v = M-m_i	v^2
1	45.66	+0.002	0.000004
2	45.68	-0.018	0.000324
3	45.66	+0.002	0.000004
4	45.65	+0.012	0.000144
sums:	182.65		0.000476

$$df = 4 - 1 = 3$$

$$M = \frac{182.65}{4} = 45.6625 = 45.662$$

$$\sigma = \sqrt{\frac{\sum v_i^2}{n-1}} = \sqrt{\frac{0.000476}{4-1}} = \pm 0.0125963 = \pm 0.012$$

$$E_M = \frac{\sigma}{\sqrt{n}} = \frac{\pm 0.0125963}{\sqrt{4}} = \pm 0.006298 = \pm 0.006$$

II. Random Errors

A. Basic Analysis

3. Comparisons

Different measurement sets can be compared for precision and accuracy.

Example: Two crew measured different angles, multiple times.

	Crew A	Crew B
Num of meas	2 D/R	4 D/R
Average angle	128°18’15”	196°02’40”
Std Dev	±0°00’12”	±0°00’14”

1 D/R = 2 angle meas.

Which crew has:

Better precision?

Better expected accuracy?

II. Random Errors

A. Basic Analysis

3. Comparisons

Different measurement sets can be compared for precision and accuracy.

Example: Two crew measured different angles, multiple times.

	Crew A	Crew B
Num of meas	2 D/R	4 D/R
Average angle	128°18’15”	196°02’40”
Std Dev	±0°00’12”	±0°00’14”

1 D/R = 2 angle meas.

Which crew has:

Better precision?

Crew A because it has a smaller standard deviation

Better expected accuracy?

Crew A: $E_M = \frac{\pm 12''}{\sqrt{2 \times 2}} = \pm 06.0''$

Crew B: $E_M = \frac{\pm 14''}{\sqrt{4 \times 2}} = \pm 04.9''$

II. Random Errors

A. Basic Analysis

4. Weights

Different quality measurements can be used together.

Use relative *weights* to give better quality measurements greater effect.

Weights can be based on

- standard deviations
- equipment specs
- equipment setup errors
- number of setups
- number of repetitions
- guesstimates (experience)

$$M_w = \frac{\sum (w_i \times m_i)}{\sum w_i}$$

$$\sigma_w = \sqrt{\frac{\sum (w_i v_i^2)}{(n-1)\left(\sum w_i\right)}}$$

II. Random Errors

A. Basic Analysis

4. Weights

Example: A distance is measured by different methods with these results.

Method	Dist, m	σ
Steel Tape	254.63	±0.21
Subtense bar	254.69	±0.16
Total Station	254.72	±0.06
sums:	*764.04*	

Unit mean: $M = \frac{764.04}{3} = 254.680$

Unit means singular or one.
Treating all the measurements the same is like using a unit weight = 1

II. Random Errors

A. Basic Analysis

4. Weights

Example: A distance is measured by different methods with these results.

Method	Dist, m	σ	w=1/σ^2	w x m
Steel Tape	254.63	±0.21	22.7	5780.1
Subtense bar	254.69	±0.16	39.1	9958.4
Total Station	254.72	±0.06	277.8	70,761.2
sums:	*764.04*		*339.6*	*86,499.7*

Unit mean: $M = \frac{764.04}{3} = 254.680$

Weighted mean: $M_w = \frac{\sum (w_i \times m_i)}{\sum w_i} = \frac{86499.7}{339.6} = 254.711$

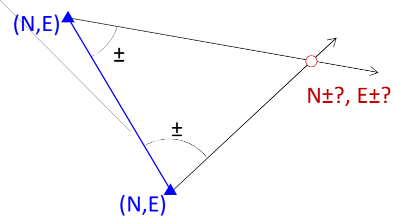
Mean is pulled toward the Total Station distance

II. Random Errors

B. Error Propagation

1. Indirect measurements

Uncertainty in desired quantity is a function of the measurement uncertainties.



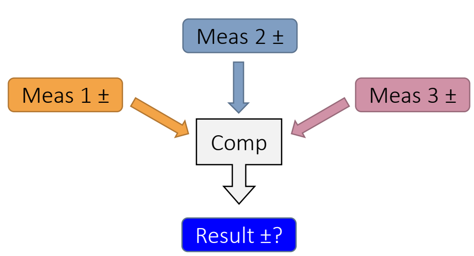
II. Random Errors

B. Error Propagation

1. Indirect measurements

The measurements are combined in some mathematical operation.

How the error propagates depends on the mathematical operation.



Because measurement errors are random, they do not propagate in a simple fashion by adding or multiplying.

II. Random Errors

B. Error Propagation

1. Indirect measurements

There are as many ways to propagate errors as there are equations to combine them.

Two common ones used in Surveying are:

a. Error of a Sum

Adding, subtracting measurements

b. Error of a Series

When the same error recurs multiple times

II. Random Errors

B. Error Propagation

2. Error of a Sum

$$E_{Sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

E_1, E_2, ..., E_n are the uncertainties in the numbers being added/subtracted.

Example:

A soil specimen is divided into three sample parts which are separately weighed. What is the error in the soil's total weight?

Soil sample	Weight
1	56.2 gr ±0.15 gr
2	28.9 gr ±0.21 gr
3	41.6 gr ±0.11 gr
	127.6 gr

$$E_{Sum} = \sqrt{(\pm 0.15)^2 + (\pm 0.21)^2 + (\pm 0.11)^2}$$

$$= \pm 0.2815 = \underline{\pm 0.28}$$

II. Random Errors

B. Error Propagation

3. Error of a Series

$$E_{Series} = E\sqrt{n}$$

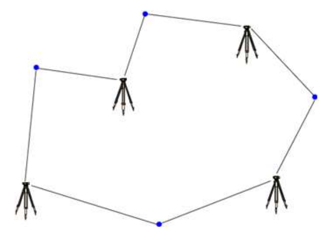
E is an expected error each time an operation is performed.

Total error is individual expected error times the square root of the chances it has to occur.

Example:

Each time they set up a level, a survey crew can determine an elevation difference to ±0.015 ft. What is their error on this loop?

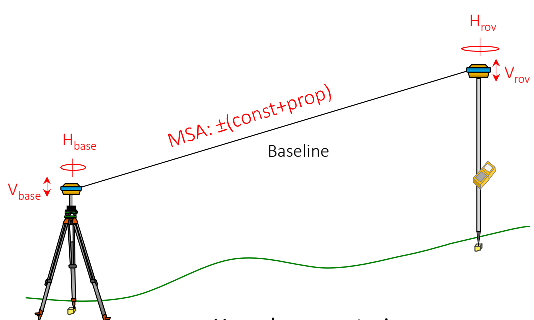
$$E_{Sum} = \pm 0.015\sqrt{4} = \underline{\pm 0.030}$$



II. Random Errors

B. Error Propagation

Example: GPS horizontal and vertical baseline components



H_{Base}: base centering error
H_{Rov}: rover centering error

V_{Base}: base HI error
V_{Rov}: rover HI error

Positioning performance

Precision Static	H: 3 mm + 0.1 ppm V: 3.5 mm + 0.4 ppm
Static/Fast Static[1]	H: 3 mm + 0.5 ppm V: 5 mm + 0.8 ppm
PPP	H: 3 cm RMS[2] V: 5 cm RMS[2] Convergence time: < 5 mins[3]
RTK[4]	H: 5 mm + 0.5 ppm V: 10 mm + 0.8 ppm
RTK, TILT Compensated	RTK + 5 mm + 0.5 mm / ° tilt Compensation up to 60°

$$E_{Sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

$$E_H = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Rov}^2}$$

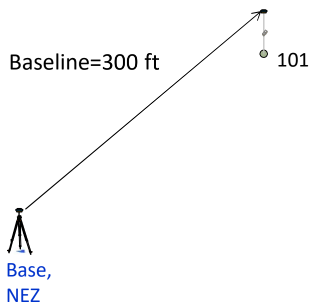
$$E_V = \sqrt{V_{Base}^2 + V_{MSA}^2 + V_{Rov}^2}$$

II. Random Errors

B. Error Propagation

Example: GPS horizontal and vertical baseline components

MSA: ±(5mm + 0.5ppm) horiz
Base centering error: E_B=±0.005ft
Rover centering error: E_R= ±0.05ft
What is baseline length error?



$$E_H = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Rov}^2}$$

$$5mm \times \frac{39.37in}{1000mm} \times \frac{1ft}{12in} = 0.0164ft$$

$$E_H = \sqrt{(0.005ft)^2 + \left(0.0164ft + \frac{300ft \times 0.5}{1,000,000}\right)^2 + (0.05ft)^2}$$

$$= \underline{\pm 0.053ft}$$

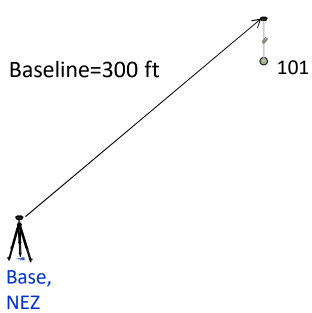
$$precision: \frac{0.053ft}{300.00ft} = \frac{1}{5360}$$

II. Random Errors

B. Error Propagation

Example: GPS horizontal and vertical baseline components

MSA: ±(10mm + 0.8ppm) vert
Base HI error: E_B=±0.005ft
Rover HI error: E_R= ±0.005ft
What is baseline elev diff error?



$$E_V = \sqrt{V_{Base}^2 + V_{MSA}^2 + V_{Rov}^2}$$

$$10mm \times \frac{39.37in}{1000mm} \times \frac{1ft}{12in} = 0.03281ft$$

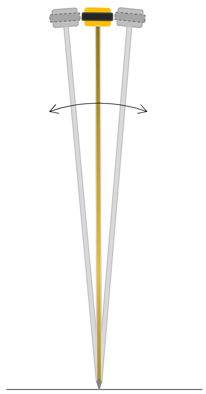
$$E_V = \sqrt{(0.005ft)^2 + \left(0.03281ft + \frac{300ft \times 0.8}{1,000,000}\right)^2 + (0.005ft)^2}$$

$$= \underline{\pm 0.034ft}$$

II. Random Errors

B. Error Propagation

Example: What about tilt compensation?



Positioning performance

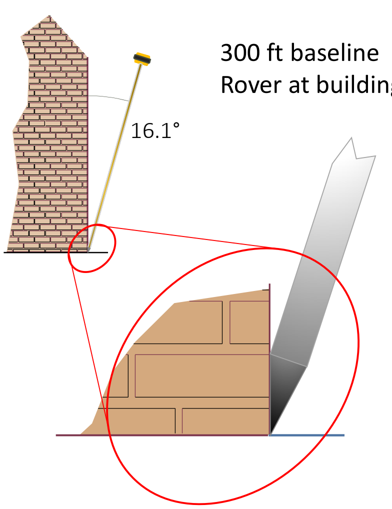
Precision Static	H: 3 mm + 0.1 ppm V: 3.5 mm + 0.4 ppm
Static/Fast Static[1]	H: 3 mm + 0.5 ppm V: 5 mm + 0.8 ppm
PPP	H: 3 cm RMS[2] V: 5 cm RMS[2] Convergence time: < 5 mins[3]
RTK[4]	H: 5 mm + 0.5 ppm V: 10 mm + 0.8 ppm
RTK, TILT Compensated	RTK + 5 mm + 0.5 mm / ° tilt Compensation up to 60°

$$E_H = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Tilt}^2}$$

II. Random Errors

B. Error Propagation

Example: What about tilt compensation?



300 ft baseline
Rover at building base

MSA: ±(5mm + 0.5ppm) horiz
Tilt: RTK + 5mm + 0.5mm/°
Base centering error: E_B=±0.005ft
What is baseline length error?

$$5mm = 0.0164ft$$

$$16.1° \times 0.5mm/° = 8.05mm \times \frac{39.37in}{1000mm} \times \frac{1ft}{12in} = 0.02641ft$$

$$E_H = \sqrt{H_{Base}^2 + H_{MSA}^2 + H_{Tilt}^2}$$

$$= \sqrt{0.005ft^2 + \left(0.0164ft + \frac{300ft \times 0.5}{1,000,000}\right)^2 + (0.0164ft + 0.02641)^2}$$

$$= \pm 0.046ft$$

$$Prec = \frac{0.046ft}{300.00ft} = \frac{1}{6520}$$

II. Random Errors

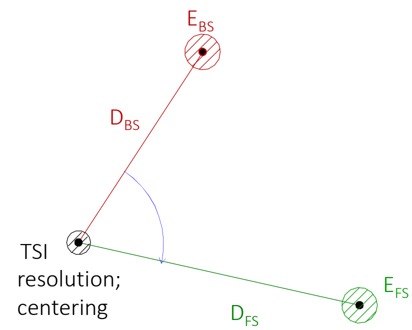
B. Error Propagation

4. Combined

Angle measurement

Angles are complicated because there are 3 points and a number of random error sources:

- Instrument angle resolution
- Instrument centering
- Target centering at BS and FS points
- Target sightings
- Distances to BS and FS points
- Number of times the angle is measured.



II. Random Errors

B. Error Propagation

4. Combined

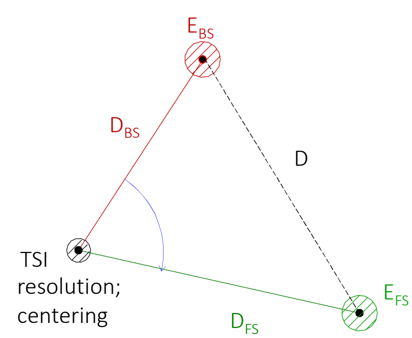
Angle measurement

TSI Point & Read $$E_{pr} = \frac{2 \times E_{DIN}}{\sqrt{n}}$$

TSI Centering $$E_{tsi} = \frac{D \times E_i}{D_{BS} D_{FS} \sqrt{2}} \times \frac{206{,}264.8 \text{ sec}}{\text{radian}}$$

Target Centering $$E_t = \sqrt{\left(\frac{E_{BS}}{D_{BS}}\right)^2 + \left(\frac{E_{FS}}{D_{FS}}\right)^2} \times \frac{206{,}264.8 \text{ sec}}{\text{radian}}$$

Angle Error $$E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$$



II. Random Errors

B. Error Propagation

4. Combined

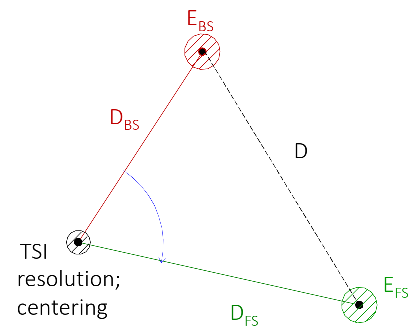
Example

A TSI with DIN spec of 2 sec;
centering error: ±0.005 ft

BS & FS centering errors: ±0.005 ft & ±0.01 ft;
BS & FS dists: 176 ft & 243 ft.

Angle was measured 2 D/R to get 123°30’10”

Many comps later....
Expected error in the angle is ±11.4”



II. Random Errors

C. Summary

Error Management

Errors can’t be eliminated but effects can be minimized

- Systematic errors
  - Procedure
  - Adjustment
  - Computation
- Random Errors
  - Appropriate equipment
  - Knowledgeable personnel
  - Careful measuring techniques
- Remaining random errors
  - Individual measurements - Analyze
  - Networks - Adjust and analyze

II. Random Errors

Now what?

- Random error will exist and must be dealt with.
  - Minimize by repetition.
  - Adjustment: Distribute part of the error back into each measurement or result.

- Adjustment should reflect measurement error behavior
  - Results might not fit perfectly
  - Appropriate weights
  - Apply a "best fit" adjustment model

III. Least Squares

A. Network
B. Adjusting a Network
C. Redundancies
D. Least Squares Adjustment
E. Pin Cushions

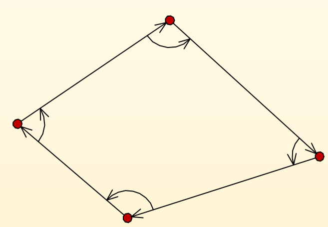
A. Network

A network consists one or more directly and/or indirectly measured quantities.

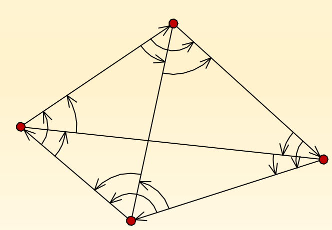
Simple network: Sometimes we know the *total* error:

$$\sum(\text{int. angles}) = (n-2)\times 180°$$

$$\sum \text{Lat} = 0 \qquad \sum \text{Dep} = 0$$



Complex network: Or there are so many measurements of varying quality that determining a “total” error is difficult or impossible.



III. Least Squares

B. Adjusting a Network

Adjustment: process of distributing error.

Simple network usually adjusted using a basic mathematical model.

Example: Each point has a single "raw" elevation.
Use an even distribution

Closure error at BM D = 814.07-824.04 = 0.03 high

Corr'n per elev = -0.03/3 = -0.01

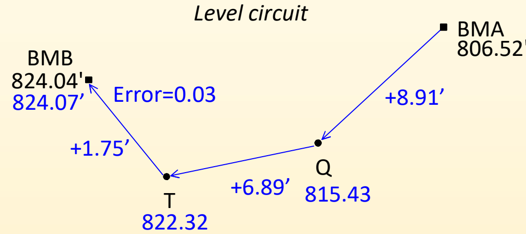
Adj $Elev_{Q}$ = 815.43+(1)(-0.01) = 815.42

Adj $Elev_{T}$ = 822.32+(2)(-0.01) = 822.30

Adj $Elev_{BMB}$ = 824.07+(3)(-0.01) = 824.04 *check*

Each point has a single adjusted elevation

Level circuit



III. Least Squares

B. Adjusting a Network

Adjustment: process of distributing error.

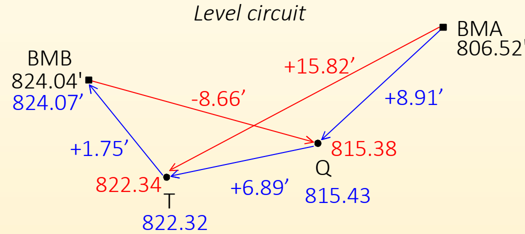
Add runs between non-adjacent points.

Points Q and T have multiple raw elevations.

How to apply simple adjustment?

Will Q & T end up with a single adjusted elevation each?

Level circuit



Have to use a "best-fit" adjustment model.

An LSA minimizes the sum of the squares of the residuals of the observations: $\Sigma(v^2)$=min

III. Least Squares

B. Adjusting a Network

Simple Adjustment

- Advantages
  - Easy
- Disadvantages
  - Treats random errors systematically
  - Can't determine quality of adjusted value

Least Squares

- Advantages
  - Models random errors better
  - Able to deal with multiple unknowns simultaneously
  - Easily incorporate redundant measurements
  - Mix different quality measurements
  - Can generate statistics for
    - overall adjustment
    - individual unknowns
- Disadvantages
  - Computation intensive
  - Statistics overload
  - Easy to misuse

III. Least Squares

C. Redundancies

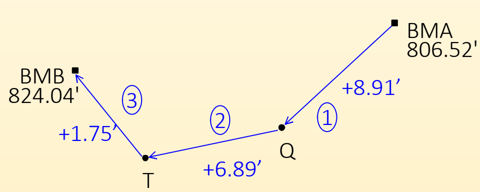
1. Vertical

Redundancy; aka Degree of freedom (df)

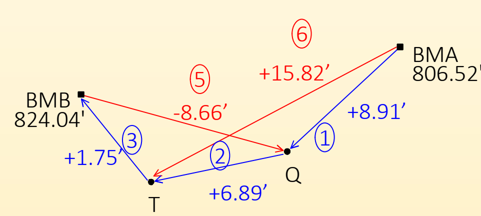
df = m-n

m: number of measurements

n: number of unknowns



m 3
n 2 Q, T
df 1



m 5
n 2 Q, T
df 3

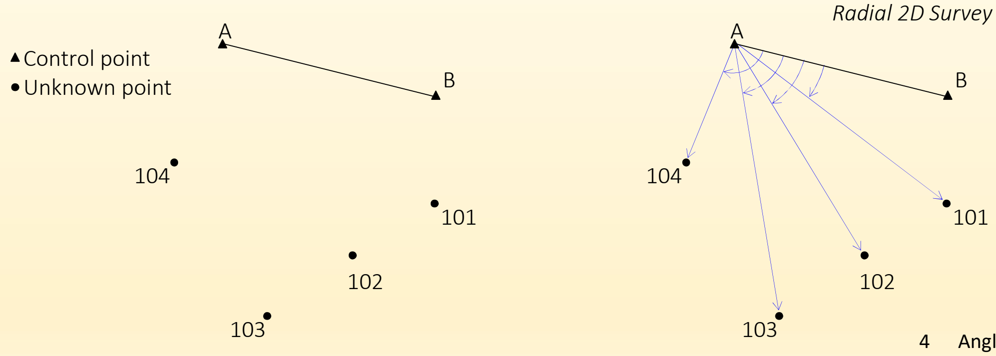
III. Least Squares

C. Redundancies

2. Horizontal

Each point in a horizontal network has two unknowns: N and E.

Radial 2D Survey



n 8 4x2 unknowns

	4	Angles
	4	Distances
m	8	Measurement
n	8	4x2 Unknowns
df	0	

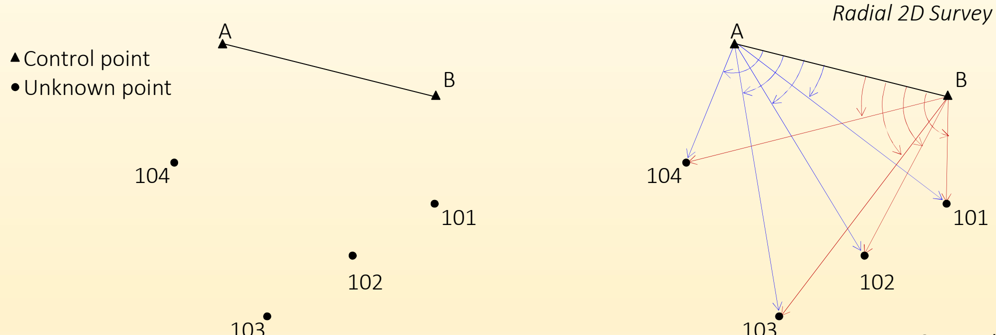
III. Least Squares

C. Redundancies

2. Horizontal

Each point in a horizontal network has two unknowns: N and E.

Radial 2D Survey



n 8 4x2 unknowns

	8	Angles
	8	Distances
m	16	Measurement
n	8	4x2 Unknowns
df	8	

III. Least Squares

D. Least Squares Adjustment

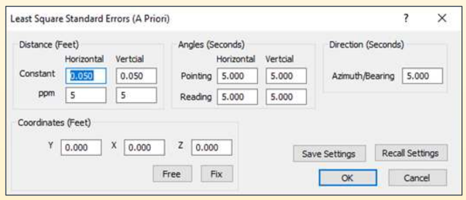
1. Fundamentals

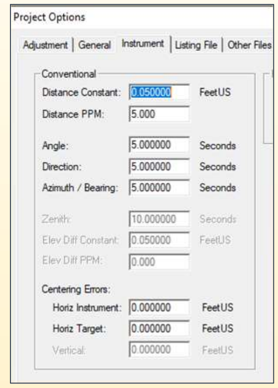
   Random errors only

   - Mistakes found and removed
   - Systematic errors compensated

   Correct weighting scheme

   - A priori estimates
     - Equipment MSA
     - Experience





An LSA best-fits measurements to control points by distributing random errors into them.

Difference between an adjusted and original measurement is its residual.

LSA minimizes the sum of the squares of the residuals: $\Sigma(v^2)$=min

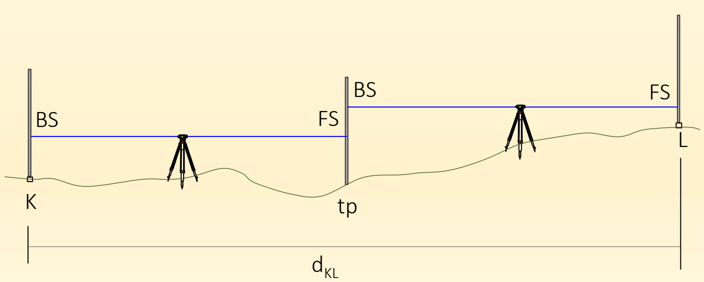
III. Least Squares

D. Least Squares Adjustment

2. Vertical

   In leveling

   - Unknowns are elevations
   - Measurements are simple addition and subtraction
     - $Elev_B = Elev_A + BS_A - FS_B$
   - Simple math means LSA is a direct solution of simultaneous equations.

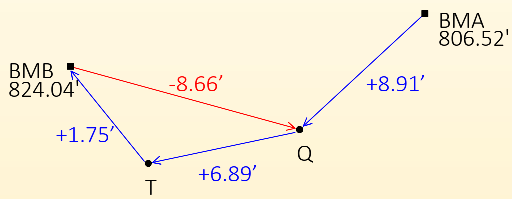


III. Least Squares

D. Least Squares Adjustment

2. Vertical

To minimize Σ(v^2), an equation including a residual must be created for each measurement.



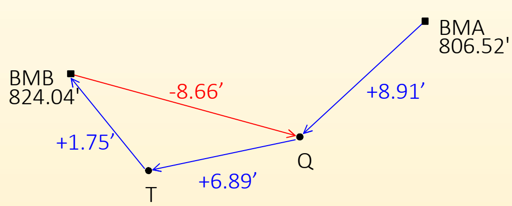
Line		
BMA-Q:	$Elev_Q = 806.52 + 8.91 + v_{aq}$	$v_{aq} = Elev_Q - 815.43$
Q-T:	$Elev_T = Elev_Q + 6.89 + v_{qt}$	$v_{qt} = Elev_T - Elev_Q - 6.89$
T-BMD:	$824.04 = Elev_T + 1.75 + v_{tb}$	$v_{tb} = 822.29 - Elev_T$
BMD-Q	$Elev_Q = 824.04 - 8.66 + v_{bq}$	$v_{bq} = Elev_Q - 815.38$

III. Least Squares

D. Least Squares Adjustment

2. Vertical

To minimize Σ(v^2), an equation including a residual must be created for each measurement.



Square each residual equation and add them

$$F = \sum_1^n v_i^2 = \left(Elev_Q - 815.43\right)^2 + \left(Elev_T - Elev_Q - 6.89\right)^2 + \left(822.29 - Elev_T\right)^2 + \left(Elev_Q - 815.38\right)^2$$

The function must be minimized for each unknown.
Take the partial derivative with respect to each unknown elevation and set equal to 0.

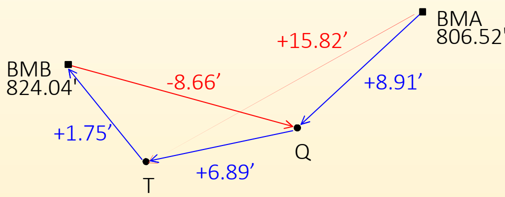
$$\frac{\partial F}{\partial Elev_Q} = 0.000 \qquad \frac{\partial F}{\partial Elev_T} = 0.000$$

III. Least Squares

D. Least Squares Adjustment

2. Vertical

To minimize Σ(v^2), an equation including a residual must be created for each measurement.



Square each residual equation and add them

$$F = \sum_{1}^{n} v_i^2 = \left(Elev_Q - 815.43\right)^2 + \left(Elev_T - Elev_Q - 6.89\right)^2 + \left(822.29 - Elev_T\right)^2 + \left(Elev_Q - 815.38\right)^2 + \left(822.34 - Elev_T\right)^2$$

The function must be minimized for each unknown.
Take the partial derivative with respect to each unknown elevation and set equal to 0.

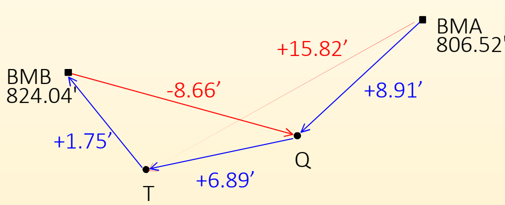
$$\frac{\partial F}{\partial Elev_Q} = 0.000 \qquad \frac{\partial F}{\partial Elev_T} = 0.000$$

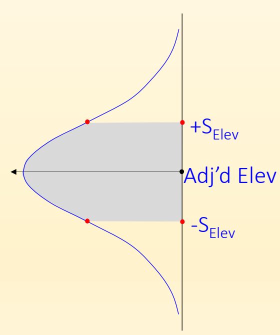
III. Least Squares

D. Least Squares Adjustment

2. Vertical

Statistics: Once elevations are determined, residuals can be computed and then standard errors.





III. Least Squares

D. Least Squares Adjustment

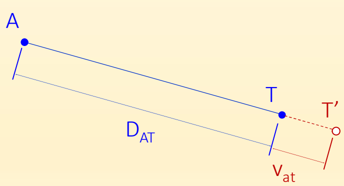
3. Horizontal

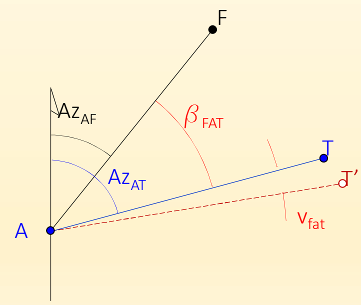
Unknowns are coordinates

Measurements are angles and distances.

Position determination requires trig which is non-linear.

Partial derivatives with trig functions is a pain.





$$v_{at} = D_{AT} - \left[\left(N_T - N_A \right)^2 + \left(E_T - E_A \right)^2 \right]^{1/2}$$

$$v_{fat} = \beta_{FAT} - \left(Az_{AT} - Az_{AF} \right) = \beta_{FAT} - \left(\tan^{-1}\left[\frac{E_T - E_A}{N_T - N_A} \right] - \tan^{-1}\left[\frac{E_F - E_A}{N_F - N_A} \right] \right)$$

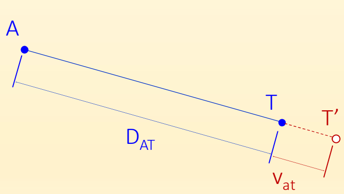
III. Least Squares

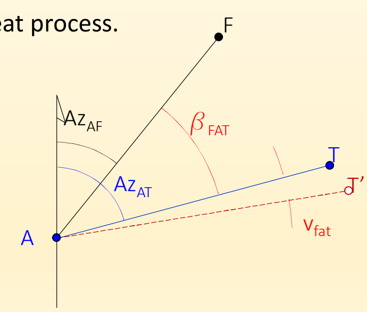
D. Least Squares Adjustment

3. Horizontal

Solution is iterative:

- Start with initial coordinate approximations
- Use minimization function to compute coordinate corrections
- If corrections are significant, apply to coordinates & repeat process.





$$v_{at} = D_{AT} - \left[\left(N_T - N_A \right)^2 + \left(E_T - E_A \right)^2 \right]^{1/2}$$

$$v_{fat} = \beta_{FAT} - \left(Az_{AT} - Az_{AF} \right) = \beta_{FAT} - \left(\tan^{-1}\left[\frac{E_T - E_A}{N_T - N_A} \right] - \tan^{-1}\left[\frac{E_F - E_A}{N_F - N_A} \right] \right)$$

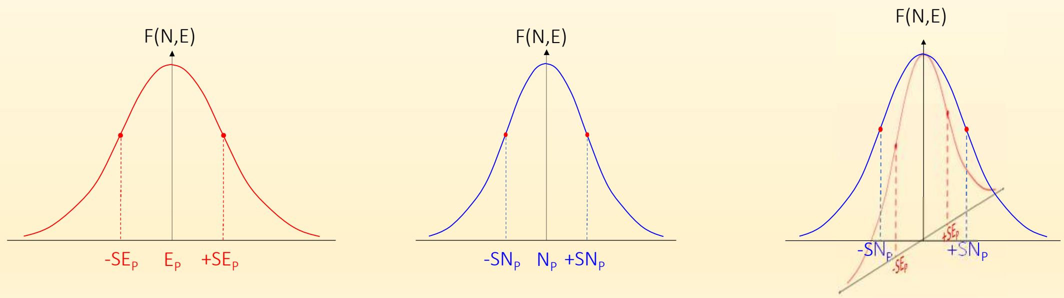
III. Least Squares

D. Least Squares Adjustment

3. Horizontal

Statistics

Each adjusted coordinate of a pair has its own standard error: S_N and S_E



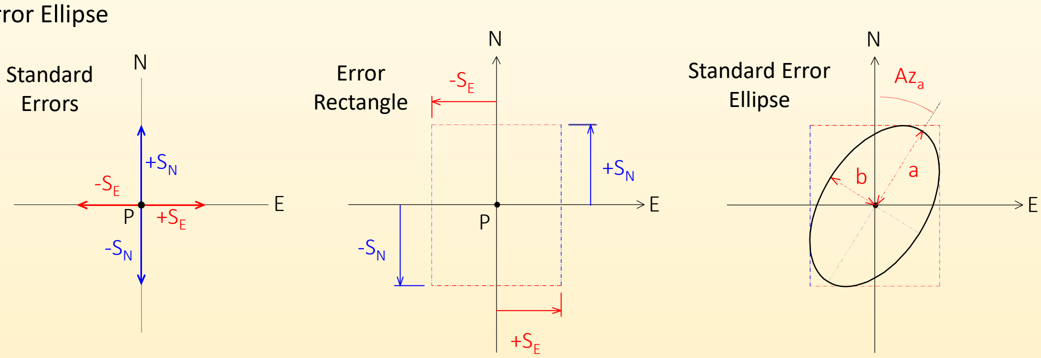
Bi-variate Distribution

III. Least Squares

D. Least Squares Adjustment

3. Horizontal

Error Ellipse



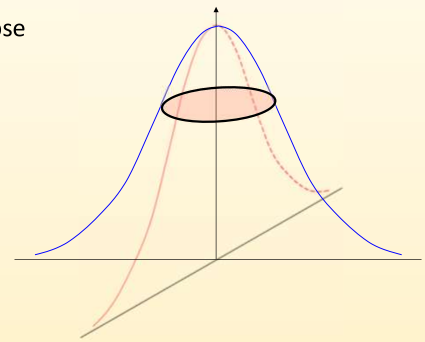
The position is weakest in the direction of the semi-major axis, a. Its direction is Az_a.
Position is strongest in the semi-minor axis, b, direction.

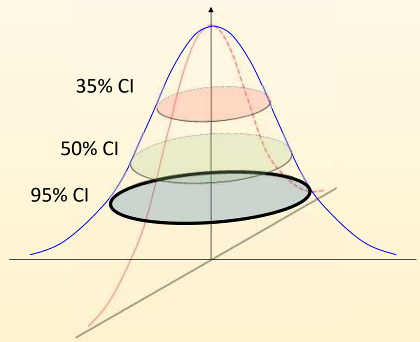
III. Least Squares

D. Least Squares Adjustment

3. Horizontal

Error Ellipse





Standard error ellipse represents approx. 33-35% of the volume under the bivariate distribution surface.
33-35% CI

95% CI error ellipse uses approx. 2.45 multiplier on the semi-major and –minor axes.
(Caution: Not all software use the same multiplier to get 95% CI)

III. Least Squares

E. Pin Cushion

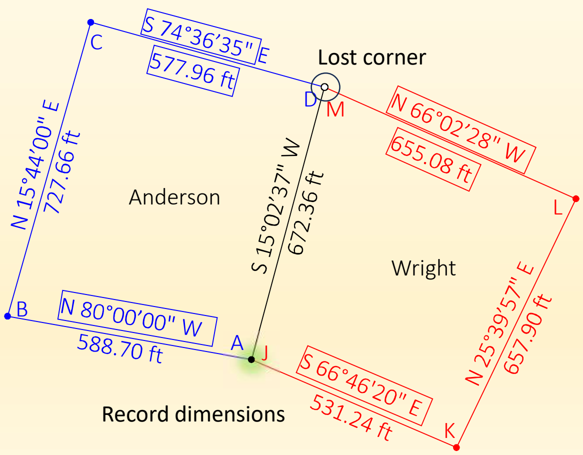
A pincushion is a corner with multiple monuments set in close proximity to each other.

Not limited to land surveyors...

III. Least Squares

E. Pin Cushion

Example: Independent surveys of two lots



Properties share common boundary.

All corners exist except common corner at D/M

2021 Jones surveys Anderson property

Starts at A, uses record bearing to B, measures to C, and uses record distance and bearing to D.

2023 Mills surveys Wright property.

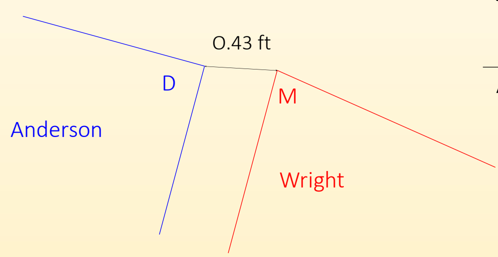
Starts at J, uses record bearing to K, measures to L, uses record distance and bearing to M

Both surveys adjusted in *StarNet* using a 95% CI

III. Least Squares

E. Pin Cushion

Example: Independent surveys of two lots



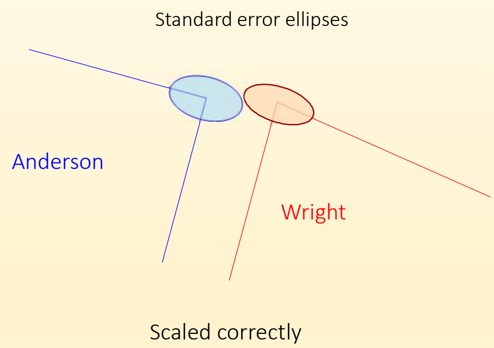
95% CI Adjustment results

Parcel	*North, ft*	*East, ft*	S_N	S_E
Anderson, D	2649.294	1174.077	0.138	0.221
Wright, M	2649.267	1174.507	0.121	0.209

III. Least Squares

E. Pin Cushion

Example: Independent surveys of two lots



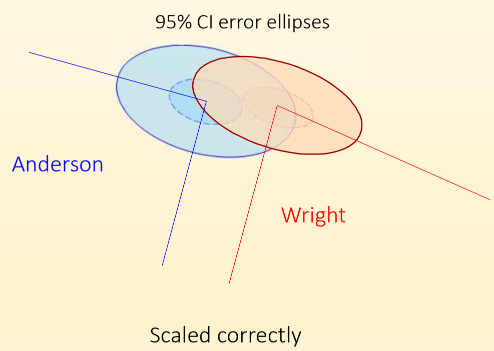
Error ellipses

Point	a	b	Az_a	
D (Anderson)	0.225	0.130	104°19’	Standard
M (Wright)	0.217	0.106	100°00’	Standard

III. Least Squares

E. Pin Cushion

Example: Independent surveys of two lots



Error ellipses

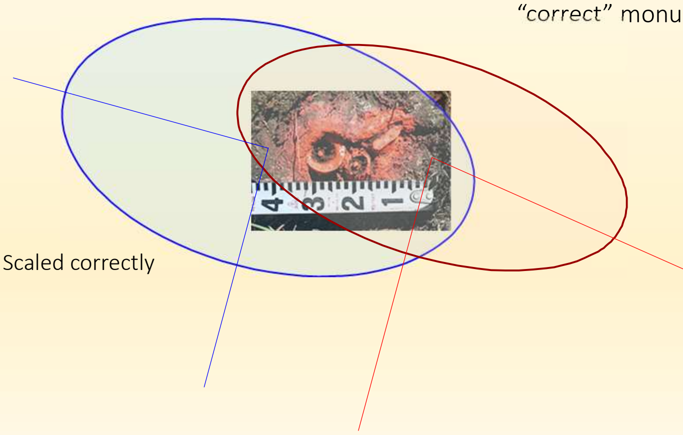
Point	a	b	Az_a	
D	0.225	0.130	104°19’	Standard
(Anderson)	0.552	0.319		95% CI
M	0.217	0.106	100°00’	Standard
(Wright)	0.531	0.260		95% CI

III. Least Squares

E. Pin Cushion

Example: Independent surveys of two lots

Should each surveyor have placed their own "correct" monument?

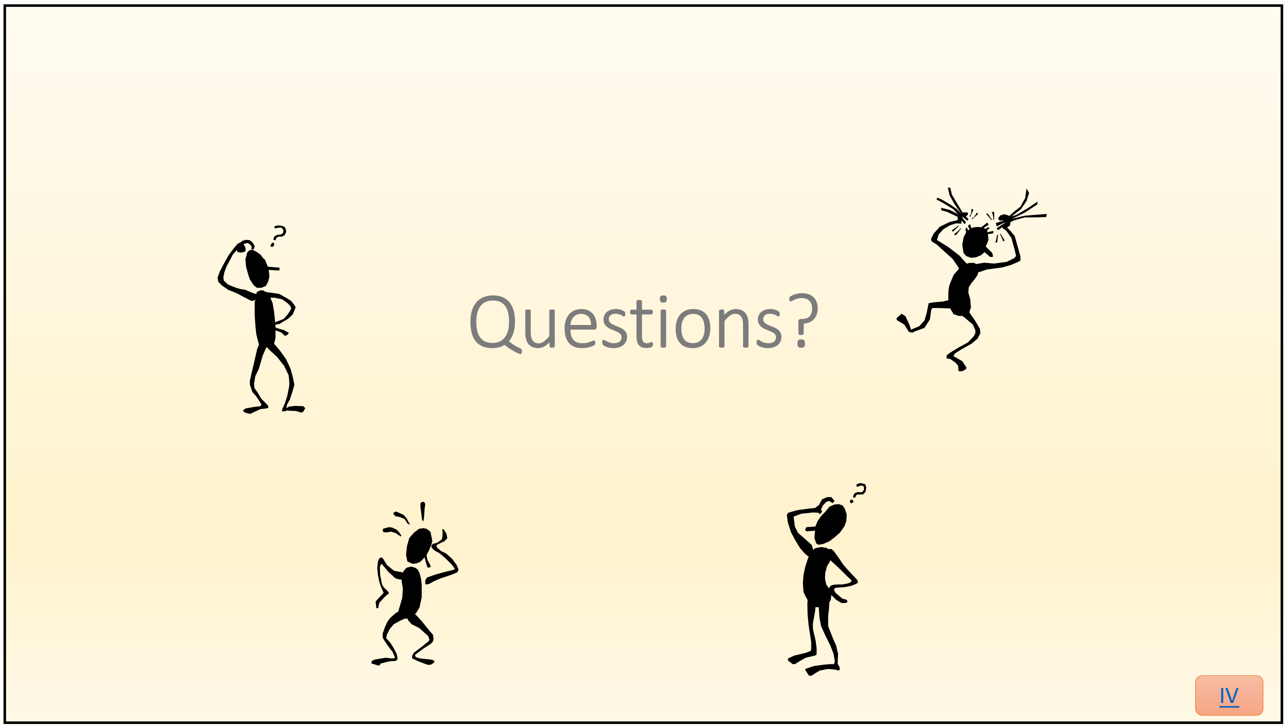


III. Least Squares

E. Pin Cushion

Understanding error behavior helps avoid creating problems like this:

Commencing at a 2" iron pipe at the west quarter corner of Section 31, T5N, R10E;
Thence S88°16'52"W, 0.30 feet to the existing east line of Section 1, T5N, R9E;
Thence S00°18'01"W, 0.01 feet along said east line of said Section 1;
Thence S00°18'01"W, 33.20 feet along said east line;
Thence N88°34'15"E, 33.78 feet to the existing east right-of-way line of STH 104, also being the point of beginning;
Thence N88°47'53"E, 803.03 feet along the existing south right-of-way line of STH 92;
Thence N88°17'20"E, 55.46 feet along said south right-of-way line;


Questions?
IV

IV. Standards and Specifications

A. Definitions

B. Control

C. Map

D. Boundary

IV. Standards and Specifications

A. Definitions

Standard: a goal or level to be achieved.

Specification: a set of rules defining procedures to achieve a standard.

Specifications are a structured and repeatable methodology.

A standard must be achieved by design not by accident.

FGCC

IV. Standards and Specifications

B. Control Surveys

1. Traditional

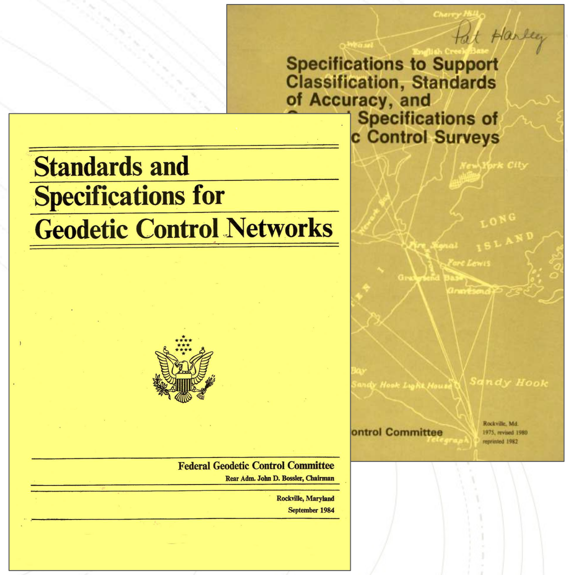
a. Responsible Agency

FGCC - *Federal Geodetic Control Committee*

Representatives from federal agencies, eg,

National Geodetic Survey
US Geological Survey
US Forest Service
US Department of Transportation
etc.

1984 Standards and Specifications.



FGCC

IV. Standards and Specifications

B. Control Surveys

1. Traditional

b. Standards

Multi-level standards depending on use.

(1) Horizontal

Standard expressed in relative terms; minimum distance precision between directly connected points

	First Order	Second Order		Third Order	
		Class I	Class II	Class I	Class II
Precision	1/100,000	1/50,000	1/20,000	1/10,000	1/5,000
Use	Primary national network, Metro area networks, Scientific studies.	Additional control to strengthen and densify primary network.	Further densification, Supplemental control.	Provide greater accessibility for lower accuracy local survey needs.	

FGCC

IV. Standards and Specifications

B. Control Surveys

1. Traditional

b. Standards

Multi-level standards depending on use.

(2) Vertical

Standard expressed in relative terms; maximum elev diff to points based on distance

	First Order		Second Order		
	Class I	Class II	Class I	Class II	Third Order
Relative Accy*	0.5 mm√K	0.7 mm√K	1.0 mm√K	1.3 mm√K	2.0 mm√K
Use	Basic framework of the National Network and of Metro area control; Extensive engr projects		Secondary Nat'l & Mertro control	Control Densification	Local control

*K: distance in km

FGCC

IV. Standards and Specifications

B. Control Surveys

1. Traditional

c. Specifications

Divided into 5 sections

Section	Purpose
Network geometry	General layout to ensure geometric strength and adequate coverage.
Instrumentation	Types and characteristics of equipment of necessary to meet requisite measurement precisions.
Calibration procedures	Nature and frequency of equipment calibration; Tolerance levels.
Field procedures	Appropriate methods of observations; measurement frequency and tolerances.
Office procedures	Data analysis, testing, and adjustments.

FGCC

IV. Standards and Specifications

B. Control Surveys

1. Traditional

c. Specifications

Can be extremely detailed depending on:

Horizontal or vertical survey

Order and class

Example: Triangulation

Angle observations for first- and second-order class I surveys can only be made at night.

Angle measurements: see table

Order / *Class*	*First*	*Second* *I*	*Second* *II*	*Third* *I*	*Third* *II*
Directions					
Number of positions	16	16	8 or 12†	4	2
Standard deviation of mean not to exceed	0.4″	0.5″	0.8″	1.2″	2.0″
Rejection limit from the mean	4″	4″	5″	5″	5″
Reciprocal Vertical Angles (along distance sight path)					
Number of independent observations direct/reverse	3	3	2	2	2
Maximum spread	10″	10″	10″	10″	20″
Maximum time interval between reciprocal angles (hr)	1	1	1	1	1

FGCC

IV. Standards and Specifications

B. Control Surveys

2. Interim

1984 Standards developed before GPS and digital levels were prevalent.

a. GPS: Because GPS measurements were more accurate than traditional angle and distances, three new horizontal orders were introduced.

Order	*Distance Precision*
AA	1:100,000,000
A	1:10,000,000
B	1:1,000,000

b. Digital levels: Vertical orders did not change but *Instrument*, *Calibration*, and *Field Procedure* sections were updated to incorporate the new technology.

~~FGCC~~ → FGDC: FGCS

IV. Standards and Specifications

B. Control Surveys

2. Interim

c. Responsible Agency

(1) FGDC: Federal Geographic Data Committee (fgdc.gov)

Established in 1990 to create policies, specifications, and standards in support of the National Spatial Data Infrastructure (NSDI).

Addresses all aspects of spatial data, not just surveying.

(2) FGCS: Federal Geodetic Control Subcommittee

FGCC becomes the FGCS under the umbrella of the FGDC.

FGDC-STD-007.2-1998

Geospatial Positioning Accuracy Standards adopted by FGCS

~~FGCC~~ → FGDC: FGCS

IV. Standards and Specifications

B. Control Surveys

3. Current Standards

FGDC-STD-007.2-1998

Federal Geographic Data Committee — FGDC-STD-007.2-1998
Draft Geospatial Positioning Accuracy Standards
Part 2: Standards for Geodetic Networks

CONTENTS

Page

~~FGCC~~ → FGDC: FGCS

IV. Standards and Specifications

B. Control Surveys

3. Current Standards

a. Position Uncertainty

Standards are defined by position uncertainty at the 95% confidence interval.

These are applicable for:

horizontal

ellipsoid height

orthometric height

Accuracy Classification	*95% CI, m*
1 mm	0.001
2	0.002
5	0.005
1 cm	0.010
2	0.020
5	0.050
1 dm	0.100
2	0.200
5	0.500
1 m	1.000
2	2.000
5	5.000
10	10.000

~~FGCC~~ → FGDC: FGCS

IV. Standards and Specifications

B. Control Surveys

3. Current Standards

b. Methodology

FGDC-STD-007.2-1998 changes how specifications are defined for a standard.

Instead of identifying equipment and specific field procedures, an *Accuracy Determination* procedure is used:

- Minimally constrained least squares adjustment to ensure correct observation weighting and freedom from blunders.

- Local and network accuracy measures computed by random error propagation to determine the provisional accuracy.

- Accuracy checked by comparing minimally constrained adjustment results against established control; must meet a 95% CI.

~~FGCC~~ »» FGDC: FGCS

IV. Standards and Specifications

B. Control Surveys

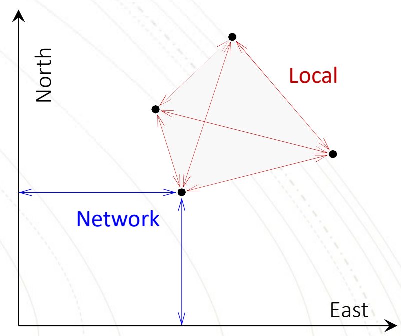
3. Current Standards

b. Methodology

Accuracies:

Local – point position uncertainty relative to other directly connected adjacent points at the 95% confidence interval.

Network - point position uncertainty relative to the geodetic datum at the 95% confidence interval.



~~FGCC~~ »» FGDC: FGCS

IV. Standards and Specifications

B. Control Surveys

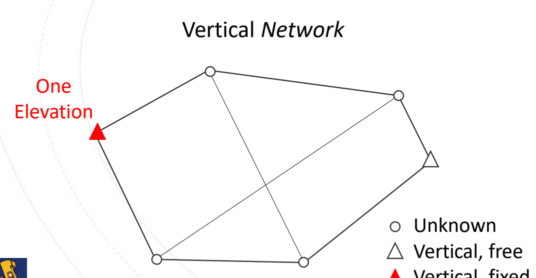
3. Current Standards

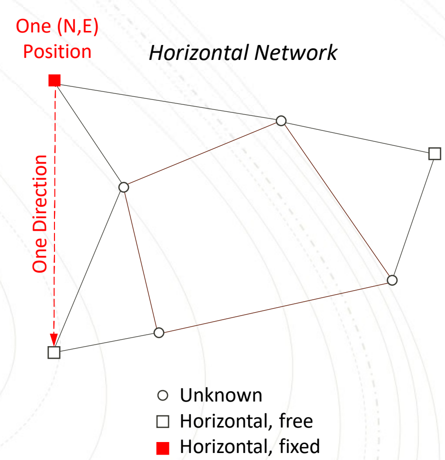
b. Methodology

Minimally constrained network adjustment.

All the measurements

Just enough control to fix the network in space



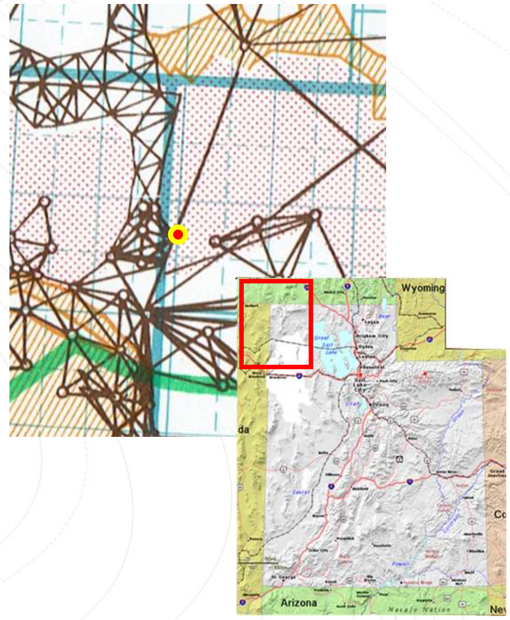


IV. Standards and Specifications

FGCC

B. Control Surveys

```
MT0717 ***********************************************************************
MT0717  DESIGNATION -  HARDISTER
MT0717  PID         -  MT0717
MT0717  STATE/COUNTY-  UT/BOX ELDER
MT0717  COUNTRY     -  US
MT0717  USGS QUAD   -  NILE SPRING (2018)
MT0717
MT0717                         *CURRENT SURVEY CONTROL
MT0717  ______________________________________________________________________
MT0717* NAD 83(1994) POSITION- 41 55 38.48001(N) 114 02 17.17156(W)   ADJUSTED
MT0717* NAVD 88 ORTHO HEIGHT -  1691.3   (meters)      5549.    (feet) VERTCON3
MT0717  ______________________________________________________________________
MT0717  GEOID HEIGHT    -         -15.225 (meters)                     GEOID18
MT0717  LAPLACE CORR    -           2.22  (seconds)                    DEFLEC18
MT0717  HORZ ORDER      -  FIRST
MT0717
MT0717.The horizontal coordinates were established by classical geodetic methods
MT0717.and adjusted by the National Geodetic Survey in June 1998.
MT0717
MT0717.The NAVD 88 height was computed by applying the VERTCON shift value to
MT0717.the NGVD 29 height (displayed under SUPERSEDED SURVEY CONTROL.)
```



IV. Standards and Specifications

~~FGCC~~ → FGDC: FGCS

B. Control Surveys

```
DH6377 ***********************************************************************
DH6377  DESIGNATION -  HPGN D CA 01 YD
DH6377  PID         -  DH6377
DH6377  STATE/COUNTY-  CA/DEL NORTE
DH6377  COUNTRY     -  US
DH6377  USGS QUAD   -  BROKEN RIB MOUNTAIN (2018)
DH6377
DH6377                         *CURRENT SURVEY CONTROL
DH6377  ______________________________________________________________________
DH6377* NAD 83(2011) POSITION- 41 58 41.85159(N) 123 43 23.38678(W)   ADJUSTED
DH6377* NAD 83(2011) ELLIP HT-    521.235 (meters)        (06/27/12)   ADJUSTED
DH6377* NAD 83(2011) EPOCH   -   2010.00
DH6377* NAVD 88 ORTHO HEIGHT -    546.7   (meters)     1794.    (feet) GPS OBS
DH6377  ______________________________________________________________________
DH6377  NAVD 88 orthometric height was determined with geoid model     GEOID03
DH6377  GEOID HEIGHT    -         -25.534 (meters)                     GEOID03
DH6377  GEOID HEIGHT    -         -25.523 (meters)                     GEOID18
DH6377  NAD 83(2011) X  - -2,636,552.944 (meters)                     COMP
DH6377  NAD 83(2011) Y  - -3,949,884.149 (meters)                     COMP
DH6377  NAD 83(2011) Z  -  4,244,160.315 (meters)                     COMP
DH6377  LAPLACE CORR    -           9.95  (seconds)                    DEFLEC18
DH6377
DH6377  Network accuracy estimates per FGDC Geospatial Positioning Accuracy
DH6377  Standards:
DH6377         FGDC (95% conf, cm)       Standard deviation (cm)      CorrNE
DH6377            Horiz  Ellip             SD_N   SD_E   SD_h      (unitless)
DH6377  -------------------------------------------------------------------
DH6377  NETWORK    1.11    1.61             0.53   0.32   0.82     -0.00516043
DH6377  -------------------------------------------------------------------
DH6377  Click here for local accuracies and other accuracy information.
DH6377
DH6377.The horizontal coordinates were established by GPS observations
DH6377.and adjusted by the National Geodetic Survey in June 2012.
```

```
DH6377 ***********************************************************************
DH6377  ACCURACIES  -  Complete network and local accuracy information.
DH6377  DESIGNATION -  HPGN D CA 01 YD
DH6377  PID         -  DH6377
DH6377
DH6377  Horiz and Ellip are the horizontal and ellipsoid height accuracies
DH6377  at the 95% confidence level per Federal Geographic Data Committee
DH6377  Geospatial Positioning Accuracy Standards.  SD_N, SD_E and SD_h are
DH6377  the standard deviations (one sigma) of the coordinates (NETWORK) or
DH6377  of the difference in the coordinates (LOCAL) in latitude, longitude
DH6377  and ellipsoid height.  CorrNE is the (unitless) correlation
DH6377  coefficient between the latitude and longitude components of either
DH6377  the coordinate (NETWORK) or coordinate difference (LOCAL).  Dist is
DH6377  the three-dimensional straight-line slope distance, in km, between
DH6377  station DH6377 and the corresponding local station.  Local stations
DH6377  are stations processed simultaneously in a session regardless of
DH6377  distance.
DH6377
DH6377  Accuracy and standard deviation values are given in cm.
DH6377
DH6377  Type/PID  Horiz  Ellip  Dist(km)  SD_N   SD_E   SD_h         CorrNE
DH6377
DH6377  -------------------------------------------------------------------
DH6377  NETWORK    1.11   1.61             0.53   0.32   0.82     -0.00516043
DH6377  -------------------------------------------------------------------
DH6377  LOCAL (003 points):
DH6377  MX1295     1.11   1.57     5.75    0.53   0.32   0.80     +0.06841477
DH6377  DH6374     1.25   1.69     9.66    0.60   0.35   0.86     -0.05156330
DH6377  NZ1330     1.06   1.43    14.43    0.51   0.30   0.73     +0.00010483
DH6377
DH6377  MEDIAN     1.11   1.57     9.66
DH6377  -------------------------------------------------------------------
```

IV. Standards and Specifications

C. Map Accuracy

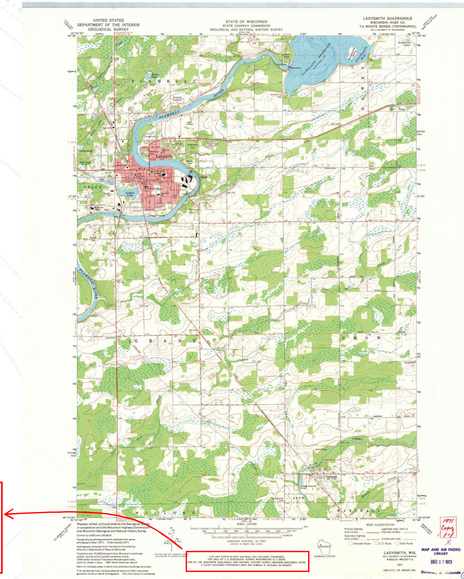
1. US National Map Accuracy Standards (USNMAS)

Date back to 1937; latest version is 1947

Purposes:

To assure the graphic accuracy of publicly funded maps.

Ensure efficient accurate data exchange between federal agencies.



THIS MAP COMPLIES WITH NATIONAL MAP ACCURACY STANDARDS
FOR SALE BY U. S. GEOLOGICAL SURVEY, WASHINGTON, D. C. 20242
AND BY THE WISCONSIN GEOLOGICAL AND NATURAL HISTORY SURVEY, MADISON, WISCONSIN 53706
A FOLDER DESCRIBING TOPOGRAPHIC MAPS AND SYMBOLS IS AVAILABLE ON REQUEST

IV. Standards and Specifications

C. Map Accuracy

1. US National Map Accuracy Standards (USNMAS)

a. Horizontal

Error of ≤10% of pts tested on map must be within:

1/30 inch at scales > 1:20,000,

1/50 inch at scales < 1:20,000

Points meas'd at publication scale.

Only for well-defined points.

b. Vertical

≤10% of elevs tested can exceed one-half the contour interval.

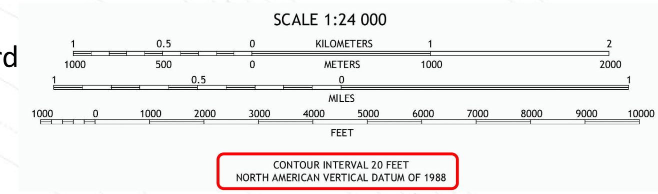
Vertical error can be compensated by allowable horiz error for the map scale

IV. Standards and Specifications

C. Map Accuracy

1. US National Map Accuracy Standard

c. Example: USGS 7.5’ topoquad



Horizontal standard

Scale 1:24,000 < 1:20,000

Use 1/50” positional accuracy.

1/50” x 20,000/1 = 400”

400” x 1’/12” = 33.3’

90% of tested points must be within ±33.3’ of their map position.

Vertical standard

CI = 20’

1/2 x 20’ = 10’

90% of tested points must be within ±10’ of their vertical map position.

IV. Standards and Specifications

C. Map Accuracy

2. FGDC

FGDC-STD-007.3-1998

National Standard for Spatial Data Accuracy (NSSDA), 1998

Uses Root Mean Squares Errors (RMSE) to determine positional accuracy at 95% CI.

RMS: square root of the average of the squared discrepancies.

e, n, z	Positions from map or digital data
E, N, Z	Positions from survey measurements
dE, dN, dZ	Discrepancy between map and survey positions
	dE = e – E
	dN = n – N
	dZ = z – Z

IV. Standards and Specifications

C. Map Accuracy

2. FGDC

Lotsa equations

Mean difference $MPV_E = \frac{\Sigma(dE_i)}{n}$ $MPV_N = \frac{\Sigma(dN_i)}{n}$ $MPV_Z = \frac{\Sigma(dZ_i)}{n}$

Std Dev difference $\sigma_E = \sqrt{\frac{\Sigma(dE_i - MPV_E)^2}{n-1}}$ $\sigma_N = \sqrt{\frac{\Sigma(dN_i - MPV_N)^2}{n-1}}$ $\sigma_Z = \sqrt{\frac{\Sigma(dZ_i - MPV_Z)^2}{n-1}}$

RMSE $RMSE_E = \sqrt{\frac{\Sigma(dE_i)^2}{n}}$ $RMSE_N = \sqrt{\frac{\Sigma(dE_i)^2}{n}}$ $RMSE_Z = \sqrt{\frac{\Sigma(dZ_i)^2}{n}}$

Vertical RMSE

Horiz RMSE $RMSE_R = \sqrt{RSME_E{}^2 + RSME_N{}^2}$

95% CI $RMSE_R = 1.7308 \times RMSE_R$ $RMSE_Z = 1.9600 \times RMSE_Z$

IV. Standards and Specifications

C. Map Accuracy

2. FGDC

Example

	Map			Survey					
Pt	e	n	z	E	N	Z	dE	dN	dZ
GCP1	3,584.394	7,449.934	477.127	3,584.534	7,450.004	477.198	-0.140	-0.070	-0.071
GCP2	3,872.190	12,939.180	412.406	3,872.290	12,939.280	412.396	-0.100	-0.100	0.010
GCP3	3,893.089	1,979.824	487.292	3,893.072	1,979.894	487.190	0.017	-0.070	0.102
GCP4	3,927.194	16,084.129	393.591	3,927.264	16,083.979	393.691	-0.070	0.150	-0.100
GCP5	16,737.074	16,675.999	451.305	16,736.944	16,675.879	451.218	0.130	0.120	0.087
						MPV	-0.033	0.006	0.006
						Std.Dev	0.108	0.119	0.091
						RMSE	0.102	0.106	
						$RMSE_R$	0.147		0.081
						95% CI	0.255		0.160

Coordinates and elevations are in meters.

IV. Standards and Specifications

D. Boundary

1. ALTA/NSPS

American Land Title Association® (ALTA)
National Society of Professional Surveyors (NSPS)

Minimum Standard Detail Requirements
For ALTA/NSPS Land Title Surveys

MINIMUM STANDARD DETAIL REQUIREMENTS FOR ALTA/NSPS LAND TITLE SURVEYS
(Effective February 23, 2026)

ALTA American Land Title Association

NSPS

IV. Standards and Specifications

D. Boundary

1. ALTA/NSPS

Section 3: Survey Standards and Standards of Care

Sub-Section E: Measurement Standards

i. "**Relative Positional Precision**" is the accepted indicator of measurement quality on an ALTA/NSPS Land Title Survey. It is defined as the length of the semi-major axis, expressed in meters or feet, of the error ellipse of the line connecting the monuments or witnesses marking adjacent boundary corners of the surveyed property at the 95 percent confidence level. Relative Positional Precision is most commonly estimated by the results of a correctly weighted least squares adjustment of the survey, or alternatively it can be estimated by the standard deviation of the distance between the monument or witness marking any boundary corner of the surveyed property and the monument or witness marking an immediately adjacent boundary corner of the surveyed property (called local accuracy) that can be computed using the full covariance matrix of the coordinate inverse between any given pair of points, understanding that Relative Positional Precision is based on the 95 percent confidence level.

IV. Standards and Specifications

D. Boundary

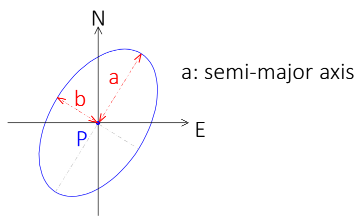
1. ALTA/NSPS

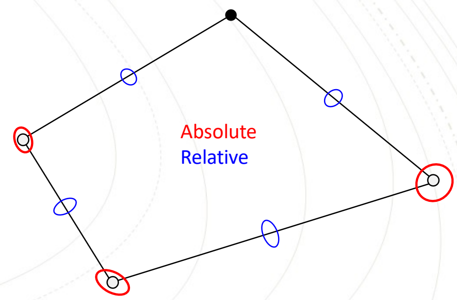
Section 3: Survey Standards and Standards of Care

Sub-Section E: Measurement Standards

Except for fixed points, each will have its own *absolute* error ellipse which an estimate of a point's position uncertainty.

A *relative* error ellipse is the uncertainty *between* points. This is local accuracy.





IV. Standards and Specifications

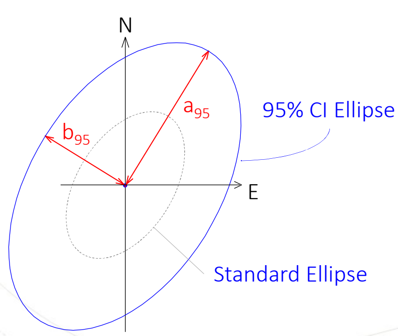
D. Boundary

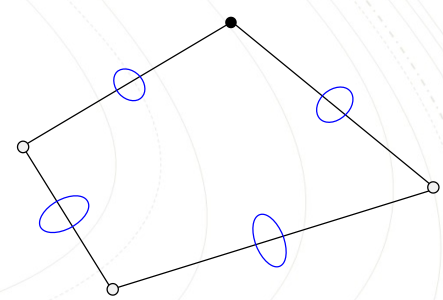
1. ALTA/NSPS

Section 3: Survey Standards and Standards of Care

Sub-Section E: Measurement Standards

Relative Position Precision (RPP) is the semi-major axis length at 95% confidence interval (CI) of a relative ellipse between adjacent property corners.





IV. Standards and Specifications

D. Boundary

1. ALTA/NSPS

Section 3: Survey Standards and Standards of Care

Sub-Section E: Measurement Standards

Maximum RPP is 2 cm (0.07 ft) plus 50 ppm between adjacent corners.

This is treated as a linear error:

$$RPP = 0.07ft + \left[D \times \frac{50}{1,000,000}\right]$$

D is the distance between points tested.

Remember: RPP is the *maximum* uncertainty; anything less is fine.

And because it is a result of random errors, it is ±.

IV. Standards and Specifications

D. Boundary

1. ALTA/NSPS

Section 3: Survey Standards and Standards of Care

Sub-Section E: Measurement Standards

Relative Position Precision

Error ellipses are determined from a properly weighted least squares adjustment of the measurements.

It is *not* a reflection of records research, evidence evaluation, or corner location.

The result of random errors in measurements.

Measurement quality is affected by:

- Equipment
- Conditions
- Procedures
- Personnel

IV. Standards and Specifications

D. Boundary

1. ALTA/NSPS

Section 3: Survey Standards and Standards of Care

Sub-Section E: Measurement Standards

"For any measurement technology or procedure used on an ALTA/NSPS Land Title Survey, the surveyor must (1) use appropriately trained personnel, (2) compensate for systematic errors, including those associated with instrument calibration, and (3) use appropriate error propagation and measurement design theory (selecting the proper instruments, geometric layouts, and field and computational procedures) to control random errors such that the maximum allowable Relative Positional Precision outlined in Section 3.E.v. below is not exceeded."

Site conditions may affect ability to achieve RPP on each line; must be noted on final plat.

IV. Standards and Specifications

D. Boundary

2. State

Each state generally has some standards or guidelines for resurveys.

Ex: Wisconsin Administrative Code *A-E 7 Minimum Standards for property Surveys*

A-E 7.01 Scope.
A-E 7.02 Definitions.
A-E 7.025 Survey report, requirements.
A-E 7.03 Boundary location.
A-E 7.04 Descriptions.
A-E 7.05 Maps.
A-E 7.06 Relative positional accuracy measurements.
A-E 7.07 Monuments.
A-E 7.08 U.S. public land survey monument record.

IV. Standards and Specifications

D. Boundary

2. State

A-E 7.06 Relative positional accuracy measurements.

(1) Measurements shall be made with instruments and methods capable of attaining the relative positional accuracy in accordance with this section.

(1m) Relative positional accuracy shall be the value expressed in feet that represents the uncertainty between points of the boundary of the parcel being surveyed due to random errors in measurements at a 95 percent confidence level.

(3) The maximum allowable deviation in relative positional accuracy between any 2 adjacent property corners may not exceed plus or minus 0.13 foot plus 100 parts per million.

(4) Any closed traverse depicted on a property survey map shall have a latitude and departure closure ratio of less than 1 in 3,000.

(5) Bearings or angles on any property survey map shall be shown to at least the nearest 30 seconds. Distances shall be shown to the nearest 1/100th foot.

IV. Standards and Specifications

A. Definitions

B. Control

C. Map

D. Boundary

Questions?