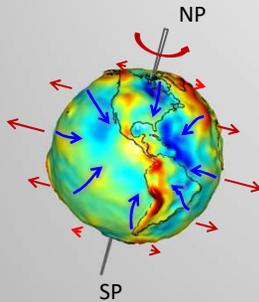


Ground and Grid Simple, Right?



LSAW Conference 2026

27 Feb 2026

Jerry Mahun, PLS

Thrice-retired, Working on my Fourth

jerry.mahun@gmail.com

<https://jerrymahun.com>



Learning Objectives

Mathematically modeling the Earth
Mechanics of a creating a grid system
Moving positions from 3D Earth to 2D grid
Formal Coordinate Systems
Controlling or compensating distortions
PLSS and Grid systems

- I. Earth Models
- II. Spatial Systems
- III. Grid Mechanics
- IV. Formal Coordinate Systems
- V. Ground and Grid
- VI. Grids and PLSS Lost Corners

**Grid and Ground
Simple, Right?**

©2020 Jerry Mahun



I. Earth Models

I. Earth Models



A. Physical Earth - Ground

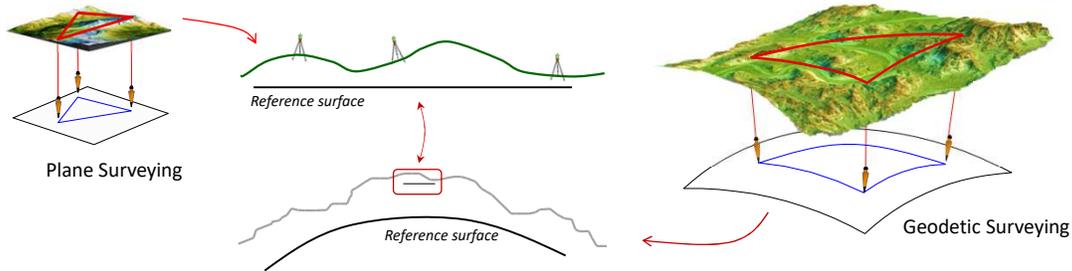
The surface on which we measure: Not mathematical.

Plane Surveying: Small areas, can assume a flat reference system

Simple trig, Plane coord system

Geodetic Surveying: Larger areas, must account for earth's shape and dynamics

Curved reference, more complicated math



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I. Earth Models



B. Geoid

Gravitational force = $f(\text{mass, distance})$

Earth: non-homogeneous; mass anomalies

⇒ Lines of gravity are neither parallel nor straight.

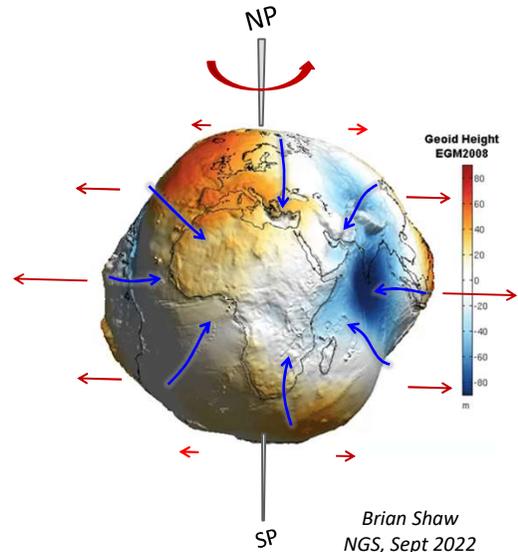
Centrifugal force = $f(\text{rotation, distance})$

Minimum at Poles ($\text{dist}=0$),

Maximum at Equator ($\text{dist}=\text{RE}$)

Gravity = Gravitational+Centrifugal forces

Oblate spheroid: Flattened at poles, enlarged around Equator



Brian Shaw
NGS, Sept 2022

Slide 6/63

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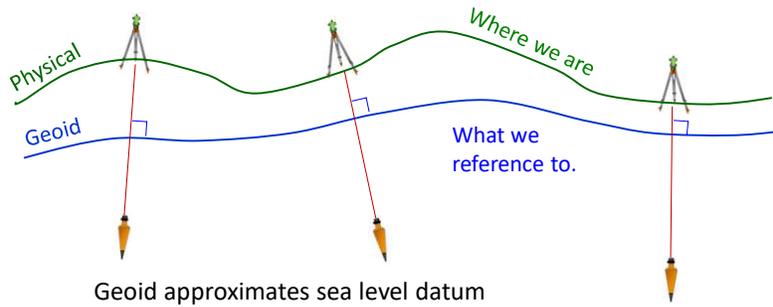
I. Earth Models



B. Geoid

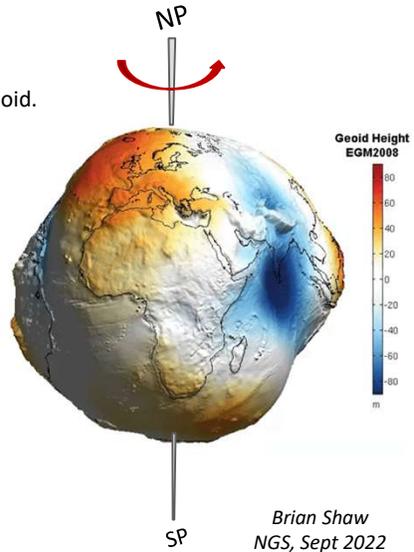
Gravity is perpendicular to the geoid

Centering a bubble or using a plumb bob orients equipment to the geoid.



Geoid approximates sea level datum

Connected to physical Earth by elevations - orthometric heights



Brian Shaw
NGS, Sept 2022

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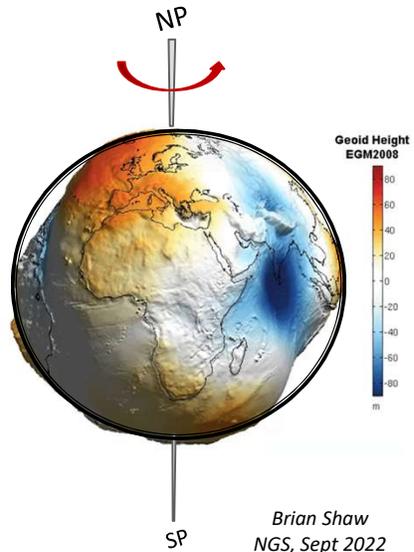
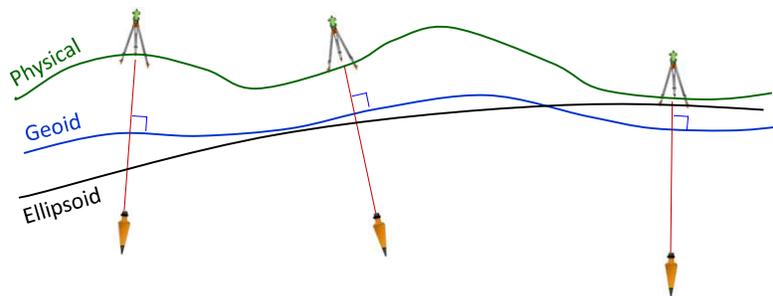
I. Earth Models



C. Ellipsoid

Mathematical 3D surface that is fit to geoid

Doesn't fit exactly; compromises



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NGS, Sept 2022

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I. Earth Models



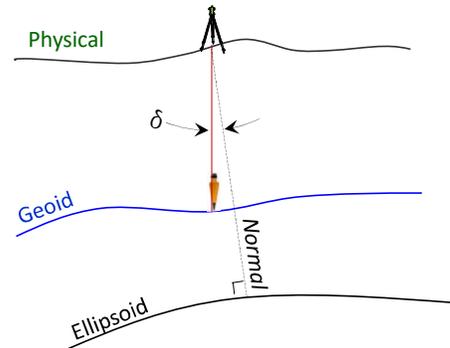
C. Ellipsoid

Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

Skewness - Deflection of the vertical, δ

Angle between directions of gravity and ellipsoid *normal*.

A normal is a line from the observer's position perpendicular to the ellipsoid



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I. Earth Models



C. Ellipsoid

Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation.

Vertical separation

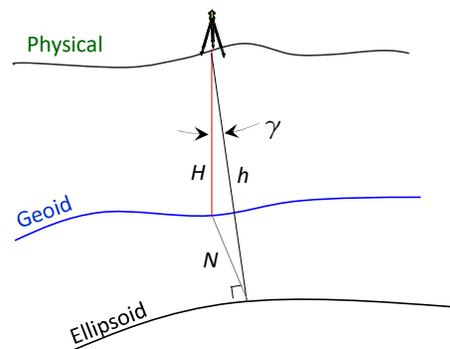
Heights between the surfaces

H - Orthometric: geoid to ground

N - Geoid: ellipsoid to geoid

h - Ellipsoidal: ellipsoid to ground

$$h = H + N$$



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I. Earth Models



C. Ellipsoid

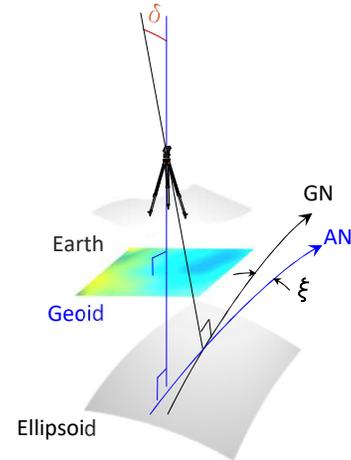
Geoid - Ellipsoid fit at a point is a function of Skewness and Vertical separation. ☆

LaPlace Corr'n, ξ

The component of δ that relates Geodetic N (GN) and Astronomic N (AN) at a particular latitude.

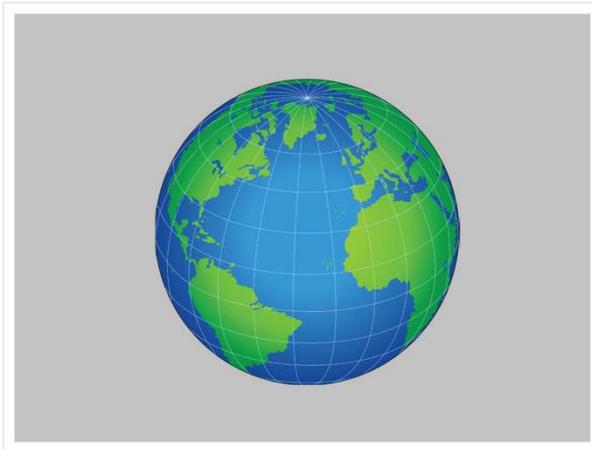
Was a common correction applied when performing astro obs for meridian determination.

See Sec 2-27 & -28, *2009 Manual*



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II. Spatial Systems

II. Spatial Systems



A. Three-Dimensional

1. Geodetic Coordinates

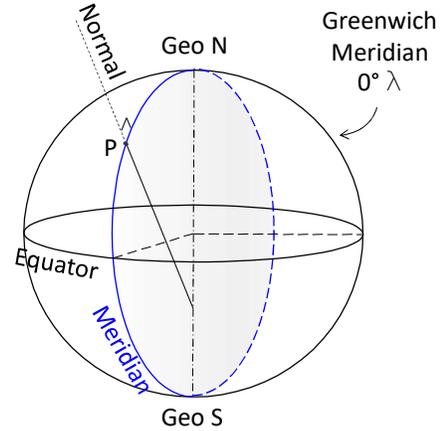
Reference defined by ellipsoid and fit.

NAD 83 - GRS 80 fit to Earth's mass center.

NAD 27 - Clarke 1866 fit to Meades Ranch, KS

Meridian An elliptical section containing the normal and semi-minor axes.
Defines Geodetic N at a point.

Geodetic meridians converge.



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II. Spatial Systems



A. Three-Dimensional

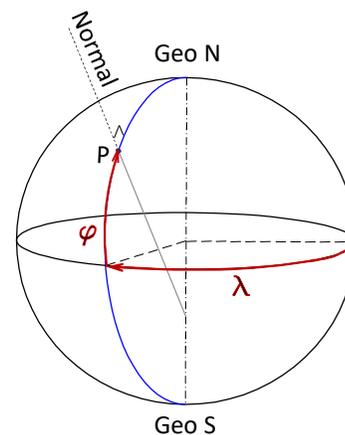
1. Geodetic Coordinates

Longitude (λ) - Angle in Equatorial plane E or W from Greenwich Meridian

0° - 180° W; 0° - 180° E

Latitude (Φ) - Angle in meridian N or S of the Equator to the normal

0° - 90° N; 0° - 90° S



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II. Spatial Systems



A. Three-Dimensional

1. Geodetic Coordinates

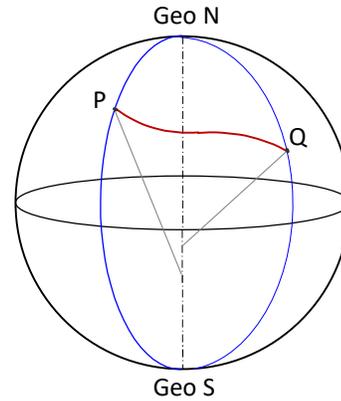
Disadvantages:

Positions are expressed in angular values

Distances are in angular values

Elliptical geometry

Shortest distance between two points is a *geodesic* - shallow s-shape curve.



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II. Spatial Systems



A. Three-Dimensional

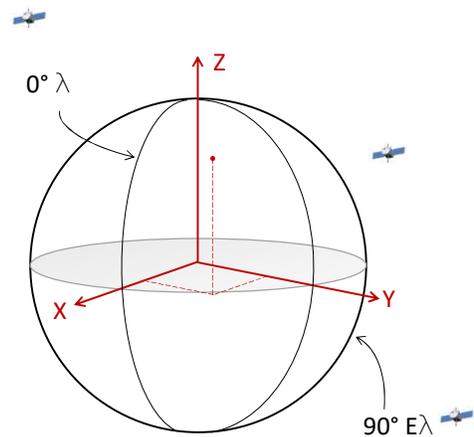
2. Terrestrial Coordinate System - TCS

Three axis rectangular system

Origin at Earth's mass center

Coordinates are linear values

Used for satellite positioning



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II. Spatial Systems

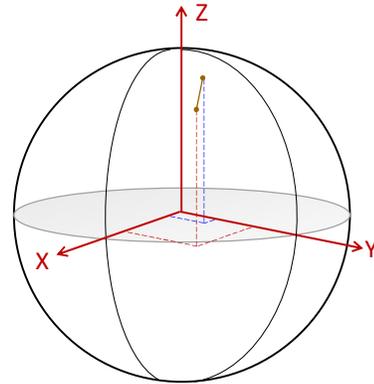


A. Three-Dimensional

2. Terrestrial Coordinate System - TCS

Disadvantages:

- Huge coordinate values.
- Negative coordinates
- No "up" (vertical direction)
 - Top and bottom of vertical structures have different 3D coordinates.



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II. Spatial Systems



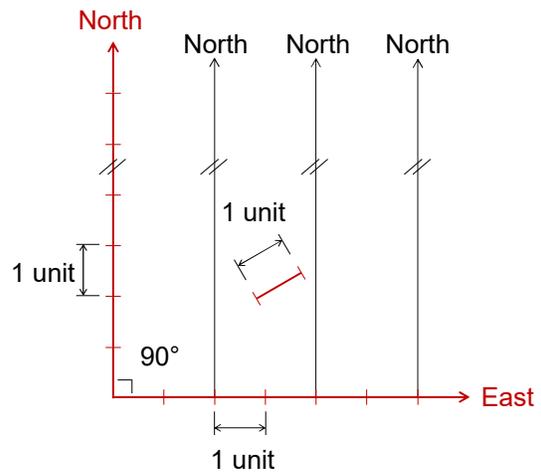
B. Two Dimensional

1. Plane

Characteristics

- a. Orthogonal
- b. Parallel north lines
- c. Uniform scale in both directions

Comps are simple.



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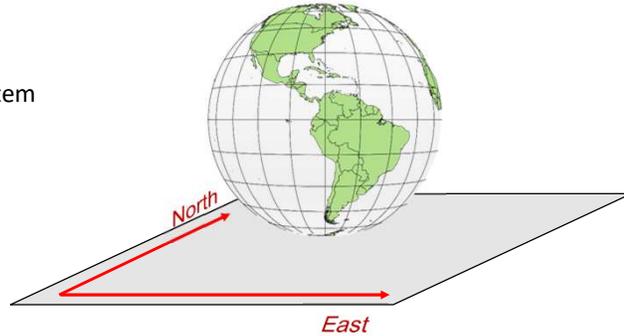
II. Spatial Systems



- B. Two Dimensional
- 2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system



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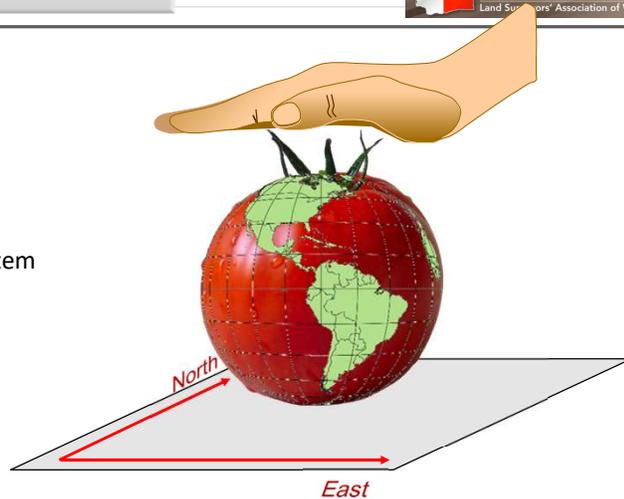
II. Spatial Systems



- B. Two Dimensional
- 2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system



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II. Spatial Systems

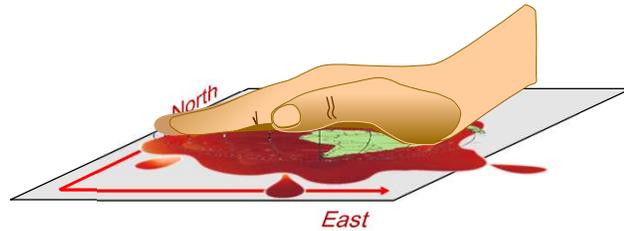


- B. Two Dimensional
- 2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system

With a direct projection we get a distorted representation



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II. Spatial Systems



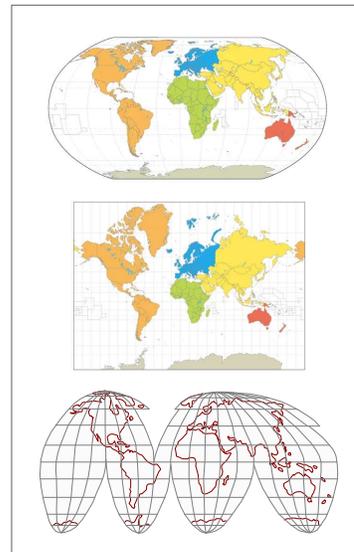
- B. Two Dimensional
- 2. Distortions

We're on a 3D irregular earth

We want to put it in a 2D mathematical system

With a direct projection we get a distorted representation

Different mathematical projections distort different ways.



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II. Spatial Systems



B. Two Dimensional

2. Distortions

Distances

Directions

Latitude and Longitude lines intersect at 90°



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II. Spatial Systems



B. Two Dimensional

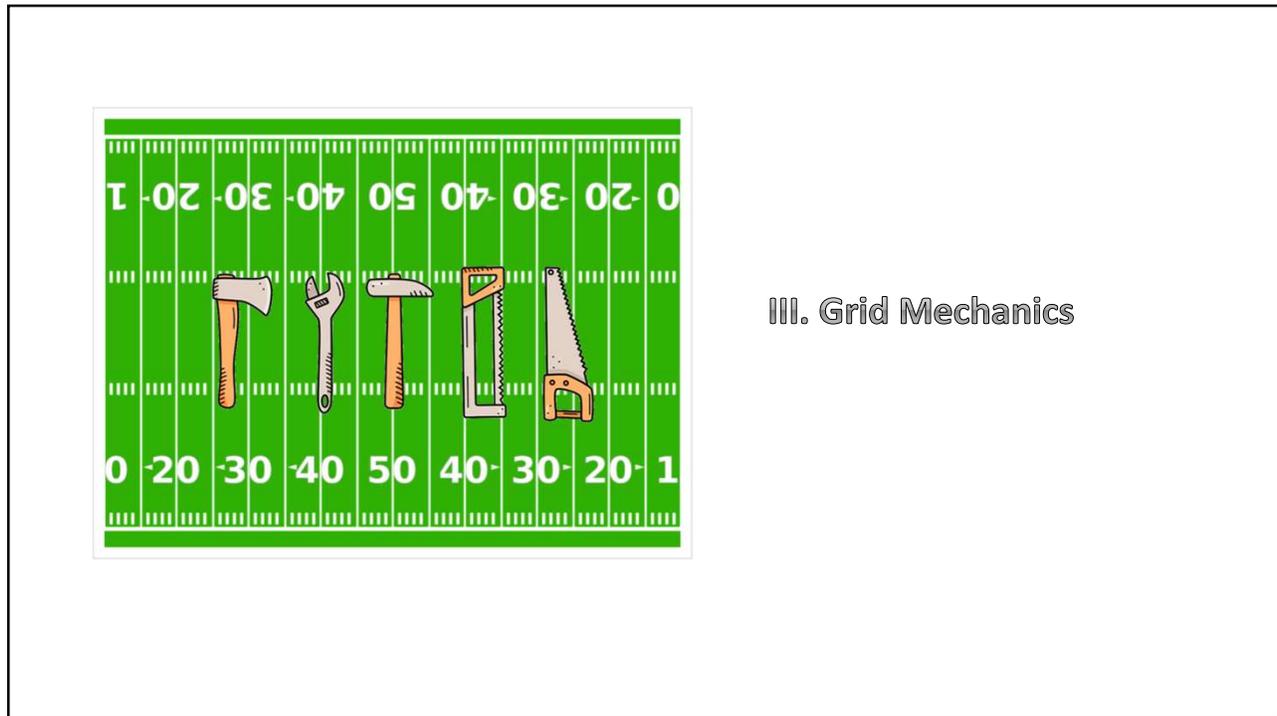
2. Distortions

The smaller the area projected,
the smaller the distortions.



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III. Grid Mechanics

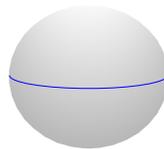
III. Grid Mechanics



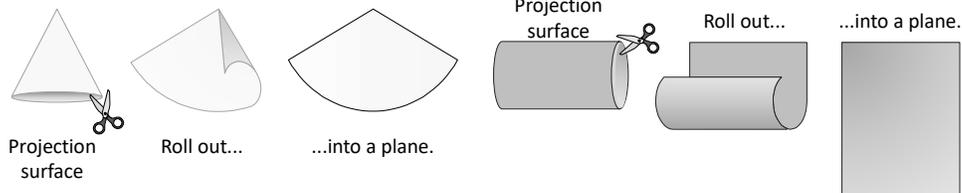
A. Projection Surfaces

To control or compensate distortions, we must project from a 3D mathematical surface to another mathematical surface that can be developed into a plane without introducing additional distortions.

The ellipsoid is the 3D surface.



A cone or cylinder are projection surfaces that can be rolled out flat into a 2D surface.



III. Grid Mechanics



A. Projection Surfaces

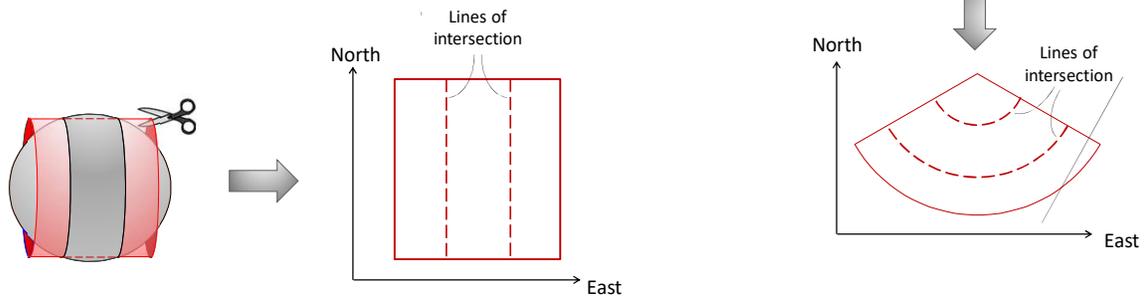
Secant projections are used

Cone or cylinder cuts through the ellipsoid.

Points are projected from the ellipsoid.

Projection is rolled out

Coordinate axes overlaid



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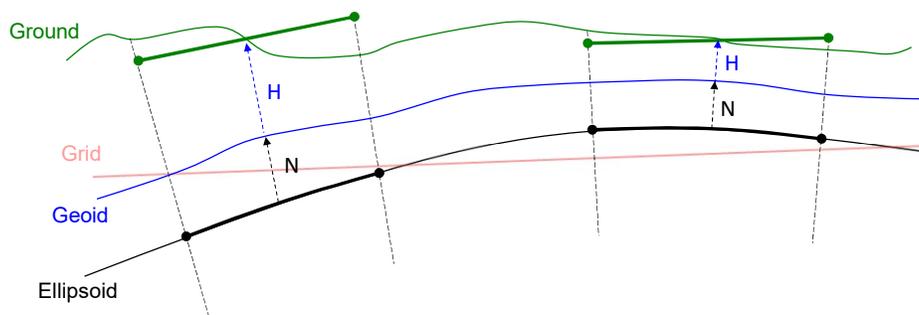
III. Grid Mechanics



B. Distance Distortion

Distances are reduced from Ground to Grid in two steps:

1. Horizontal ground to geodetic on the ellipsoid = $f(\text{orthometric, geoid heights})$



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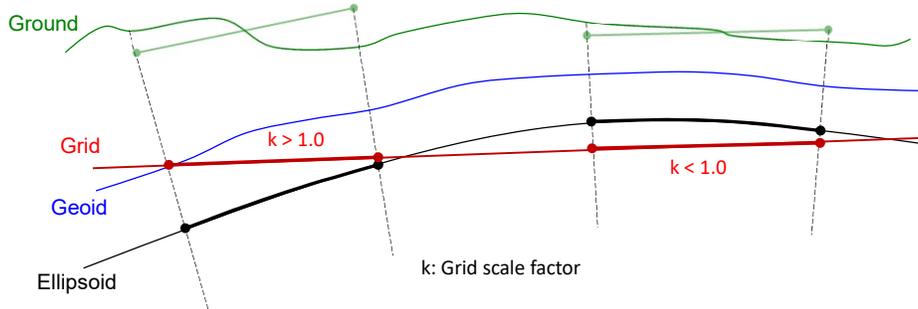
III. Grid Mechanics



B. Distance Distortion

Distances are reduced from Ground to Grid in two steps:

1. Horizontal ground to geodetic on the ellipsoid = $f(\text{orthometric, geoid heights})$
2. Geodetic to grid = $f(\text{grid scale factor})$



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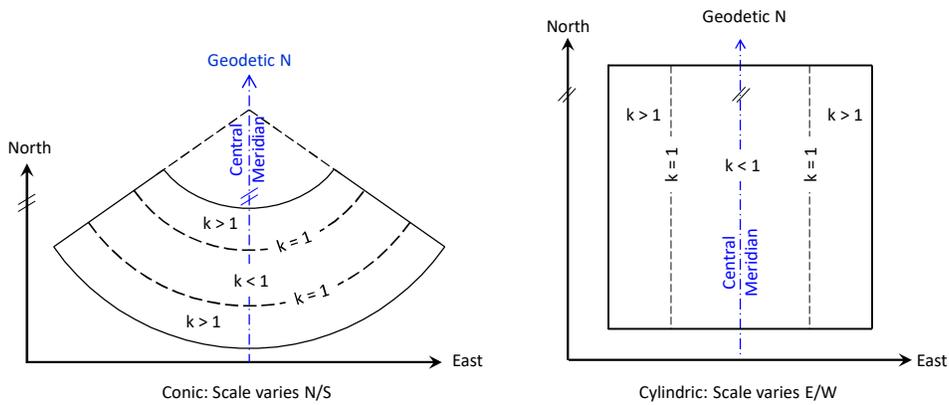
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III. Grid Mechanics



B. Distance Distortion

k: grid scale factor
Ellipsoid to grid



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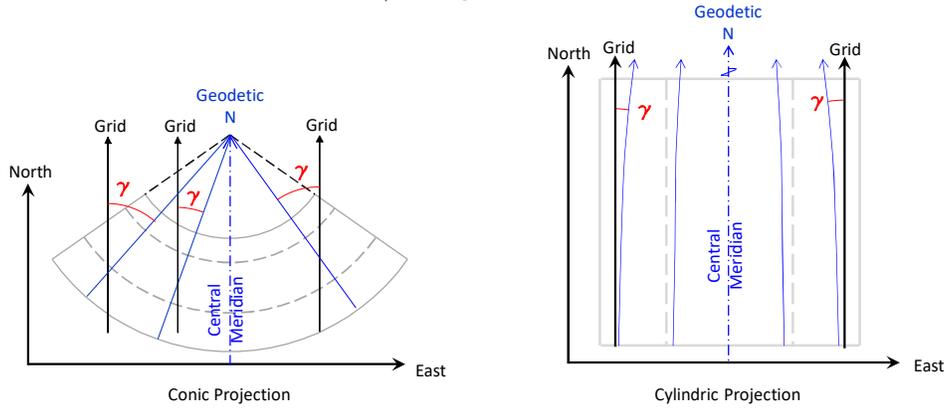
III. Grid Mechanics



C. Direction Distortion

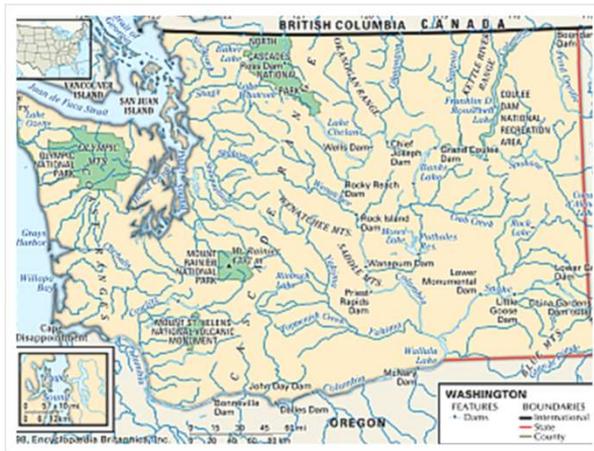
Convergence, γ , is angle between Grid and Geodetic North.

0° at CM, increases to E and to W; $\gamma=f(\text{Longitude})$



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IV. Formal Coordinate Systems

IV. Formal Coordinate Systems



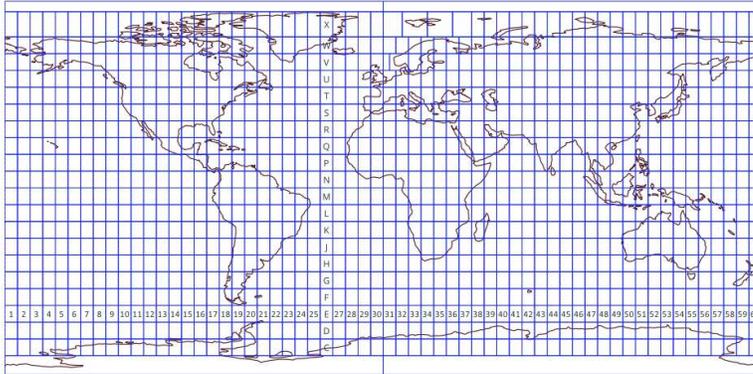
A. Universal Transverse Mercator (UTM)

60 adjacent cylindrical projections circumventing the Earth

Each projection is 6° wide and runs from 80° S Lat to 84° N Lat.

Numbered 1 to 60 from west to east and lettered C to X south to north

Max distortion: 1/2500; k=0.9996 to 1.0004



Same zones used in NAD 27 and NAD 83(xx).

Not NGS designed, but are included in NSRS and supported in NCAT.

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IV. Formal Coordinate Systems



B. State Plane Coordinates (SPC)

Designed by NGS (C&GS), included in NSRS and supported in NCAT

1. NAD 27

Development began in 1930s

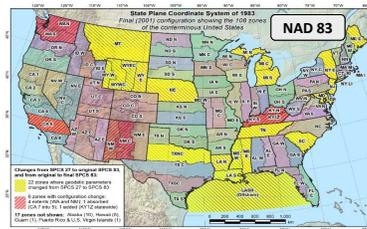
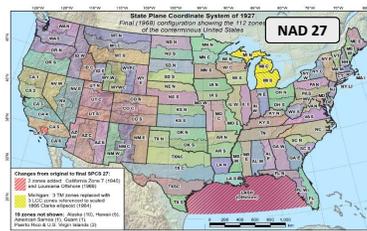
Maximum distance distortion 1/10,000 (ellipsoid to grid)

k = 0.9999 to 1.0001

→ multiple zones in most states

2. NAD 83(xx)

Some zone reshuffling



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IV. Formal Coordinate Systems



C. Current Washington Systems

1. State Plane Coordinate (SPC) system



Two conic projections
 1/10,000 Ellipsoid to Grid
 $k = 0.9999$ to 1.0001
 Central Meridian
 North $120^{\circ}50' W \lambda$
 South $120^{\circ}30' W \lambda$
 Along CM, $\gamma = 0^{\circ}00'$

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IV. Formal Coordinate Systems



C. Current Washington Systems

2. Universal Transverse Mercator (UTM)



Two cylindric projections
 1/2500 Ellipsoid to Grid
 $k = 0.9996$ to 1.0004

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V. Ground and Grid

What's the beef?

V. Ground and Grid



A. Distortion Compensation

1. Distance

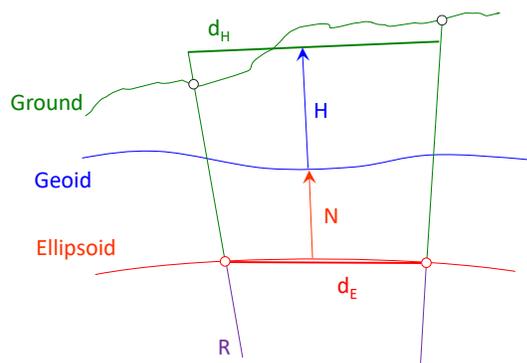
Two steps

a. Ground to ellipsoid

$$EF = \frac{R}{R + H + N}$$

$$d_E = d_H \times EF$$

- d_H Horizontal ground distance
- d_E Ellipsoidal (geodetic) distance
- EF Elevation Factor
- R Mean earth radius
- H Orthometric ht (elev)
- N Geoid height
- k Scale factor



$$R = 20,902,000 \text{ ft} = 6,371,000 \text{ m (approx.)}$$

V. Ground and Grid



A. Distortion Compensation

1. Distance

Two steps

b. Ellipsoidal to grid

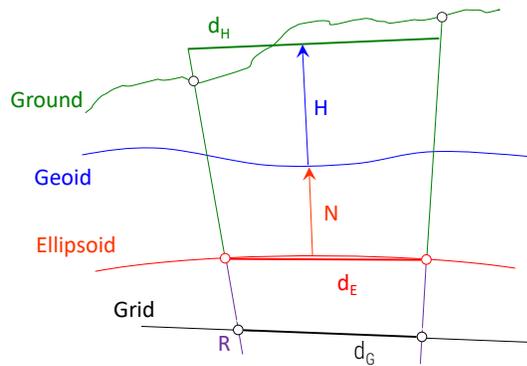
$$d_G = d_E \times k$$

d_G Grid distance
 d_E Ellipsoidal (geodetic) distance
 k Grid scale factor

c. Combined factor

$$CF = EF \times k$$

$$d_G = d_H \times CF$$



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V. Ground and Grid



A. Distortion Compensation

2. Direction

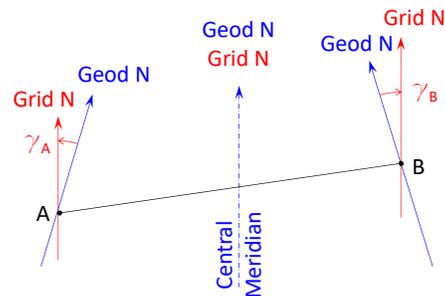
The convergence angle, γ , is **from** Geodetic N
to Grid N

It is positive (cw) East of the CM, negative
(ccw) West of the CM

To convert Geodetic (Ground) direction to Grid:

$$t = \alpha - \gamma$$

t Grid azimuth
 α Geodetic azimuth
 γ Convergence



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V. Ground and Grid



B. Reduction Elements

Where do we get the ortho and geoid heights, scale, and convergence angles?

1. NGS software (Geodetic Tool Kit):

NCAT¹

GEOIDX

Ortho heights from USGS topoquads

¹NCAT does not currently support local LDPs. When NATRF2022 is adopted, NCAT will include NGS-accepted LDPs.

The screenshot shows the 'Geodetic Tool Kit' interface with the following data in the 'Converted Coordinate' table:

Reference Frame: NAD83(2011)		SPC		UTMUSNG		XYZ (m)	
Lat-Lon-Height		SPC		UTMUSNG		XYZ (m)	
Latitude	N47° 37' 11.24269"	Zone	VIA N-4601	Zone	11	X	N/A
	N473711.24269	Northing	74,226,054 (m)	Northing (m)	5,274,180,800	Y	N/A
	47.6197896361		243,523,313 (usft)	Easting (m)	459,896,794	Z	N/A
Longitude	W117° 32' 01.31985"	Easting	747,959,117 (m)	Convergence (dms)	-00 23 39.28		
	-117.5336999583		2,453,929,202 (usft)	Scale factor	0.99961976		
Ellipsoid Height ()	Not given	Convergence (dms)	02 27 23.92	Combined factor	N/A		
		Scale factor	0.99997981	USNG	11TMNS989674180		
		Combined factor	N/A				

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V. Ground and Grid



B. Reduction Elements

2. NSRS Datasheet

AC5502	DESIGNATION	-	ARP			
AC5502	PID	-	AC5502			
AC5502	STATE/COUNTY	-	WA/SPOKANE			
AC5502	COUNTRY	-	US			
AC5502	USGS QUAD	-	FOUR LAKES (2017)			
AC5502						
AC5502						
AC5502						
AC5502						
AC5502						
AC5502*	NAD 83(2011)	POSITION-	47 37 11.24269(N) 117 32 01.31985 (W)			ADJUSTED
AC5502*	NAD 83(2011)	ELLIP HT-	700.319 (meters)		(06/27/12)	ADJUSTED
AC5502*	NAD 83(2011)	EPOCH	-	2010.00		
AC5502*	NAVD 88	ORTHO HEIGHT	-	718.71 (meters)	2358.0 (feet)	GPS OBS
AC5502						
AC5502						
AC5502	NAVD 88	orthometric height was determined with geoid model				GEOID96
AC5502	GEOID HEIGHT	-	-18.205 (meters)			GEOID96
AC5502	GEOID HEIGHT	-	-18.390 (meters)			GEOID18
AC5502	NAD 83(2011)	X	-	-1,991,234.515 (meters)		COMP
AC5502	NAD 83(2011)	Y	-	-3,819,639.008 (meters)		COMP
AC5502	NAD 83(2011)	Z	-	4,689,002.587 (meters)		COMP
AC5502	LAPLACE CORR	-	2.33 (seconds)			DEFLECI8

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V. Ground and Grid



B. Reduction Elements

2. NSRS Datasheet

```

AC5502 DESIGNATION - ARP

AC5502. The following values were computed from the NAD 83(2011) position.
AC5502
AC5502;          North      East      Units  Scale Factor  Converg.
AC5502;SPC WA N -   74,226.054  747,959.117  MT    0.99997981   +2 27 23.9
AC5502;SPC WA N -  243,523.31  2,453,929.20  sFT   0.99997981   +2 27 23.9
AC5502;UTM  11  -   5,274,180.800  459,896.794  MT    0.99961976   -0 23 39.3
AC5502
AC5502!          -  Elev Factor  x  Scale Factor =  Combined Factor
AC5502!SPC WA N -  0.99989025  x  0.99997981 =  0.99987006 } At ARP's
AC5502!UTM  11  -  0.99989025  x  0.99961976 =  0.99951005 } elevation
    
```

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V. Ground and Grid



C. Variations

1. Elevation factor, EF

$$EF = \frac{R}{R + H + N}$$

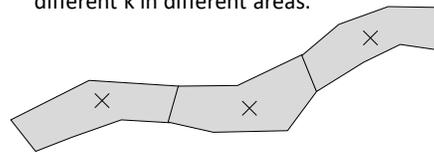
N doesn't change much so can generally use a single value over the project area.

Depending on terrain, H can be:
 project area average – use for all lines
 computed average for each line

2. Grid scale, k

For relatively small projects, a single value at project center could be used.

Larger/longer projects would require applying different k in different areas.



SPC/UTM - use approx. lat & long with NCAT to determine k.

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V. Ground and Grid



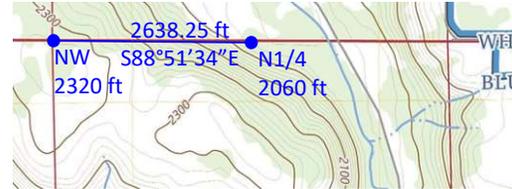
D. SPC Example

Quarter line NW-N1/4 of Sec 02 T25N R41E

Distance: 2638.25 ft

Bearing: S88°51'34"E

Determine WA SPC North zone grid distance and bearing from NW to N1/4



From topoquad

NW elev: 2320 ft

N1/4 elev: 2060 ft

Approx position of NW corner is:

47°42.1' Lat

117°36.2' W Long

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V. Ground and Grid



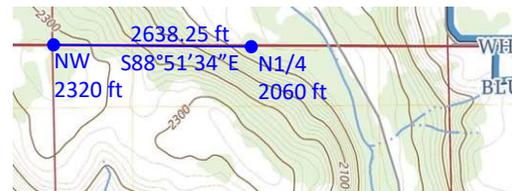
D. SPC Example

Quarter line NW-N1/4 of Sec 02 T25N R41E

Distance: 2638.25 ft

Bearing: S88°51'34"E

Determine WA SPC North zone grid distance and bearing from NW to N1/4



From topoquad

NW elev: 2320 ft

N1/4 elev: 2060 ft

Approx position of NW corner is:

47°42.1' Lat

117°36.2' W Long

From NCAT

$k = 0.99996\ 857$

$\gamma = +2^\circ 24' 15.9''$

From GEOID18

$N = -18.423\ m$

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V. Ground and Grid



D. SPC Example

1. Distance

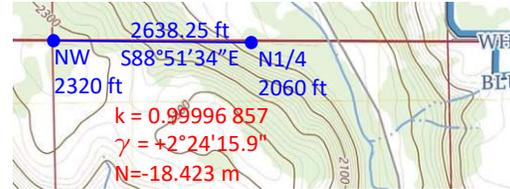
a. Ground to ellipsoid

$$H = \frac{2320 + 2060}{2} = 2190$$

$$EF = \frac{20,902,000}{20,902,000 + 2190 + (-60.44)}$$

$$= 0.99989\ 8127$$

$$d_E = 2638.25\text{ft} \times 0.99989\ 8127 = 2637.9812\text{ft}$$



$$R = 20,902,000\text{ ft}$$

$$N = -18.423\text{ m} = -60.44\text{ ft}$$

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V. Ground and Grid



D. SPC Example

1. Distance

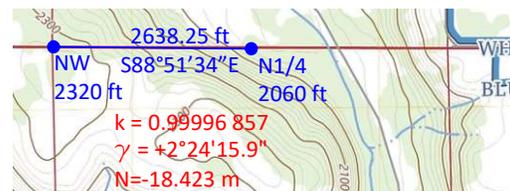
a. Ground to ellipsoid

$$H = \frac{2320 + 2060}{2} = 2190$$

$$EF = \frac{20,902,000}{20,902,000 + 2190 + (-60.44)}$$

$$= 0.99989\ 8127$$

$$d_E = 2638.25\text{ft} \times 0.99989\ 8127 = 2637.9812\text{ft}$$



b. Ellipsoid to Grid

$$d_G = 2637.9812\text{ ft} \times 0.99996\ 587$$

$$= \underline{2637.8983\text{ ft}}$$

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V. Ground and Grid



D. SPC Example

1. Distance

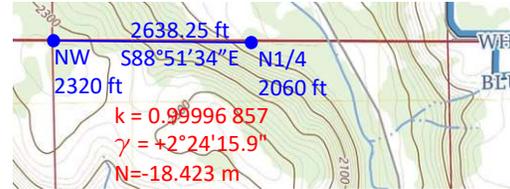
a. Ground to ellipsoid

$$H = \frac{2320 + 2060}{2} = 2190$$

$$EF = \frac{20,902,000}{20,902,000 + 2190 + (-60.44)}$$

$$= 0.99989\ 8127$$

$$d_E = 2638.25\text{ft} \times 0.99989\ 8127 = 2637.9812\text{ft}$$



b. Ellipsoid to Grid

$$d_G = 2637.9812\ \text{ft} \times 0.99996\ 587$$

$$= 2637.8983\ \text{ft}$$

$$\text{Ground to Grid distortion: } \frac{2638.25 - 2637.898}{2637.898} = \frac{0.352}{2637.898} = \frac{1}{7490}$$

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V. Ground and Grid



D. SPC Example

2. Direction

Convert bearing to azimuth

$$Az = 180^{\circ}00'00'' - 88^{\circ}11'34''$$

$$= 91^{\circ}48'26''$$

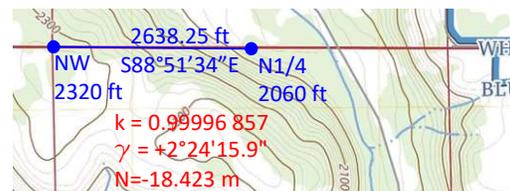
Convert to Grid Az

$$\text{Grid Az} = 91^{\circ}48'26'' - (+2^{\circ}24'16'')$$

$$= 89^{\circ}24'10''$$

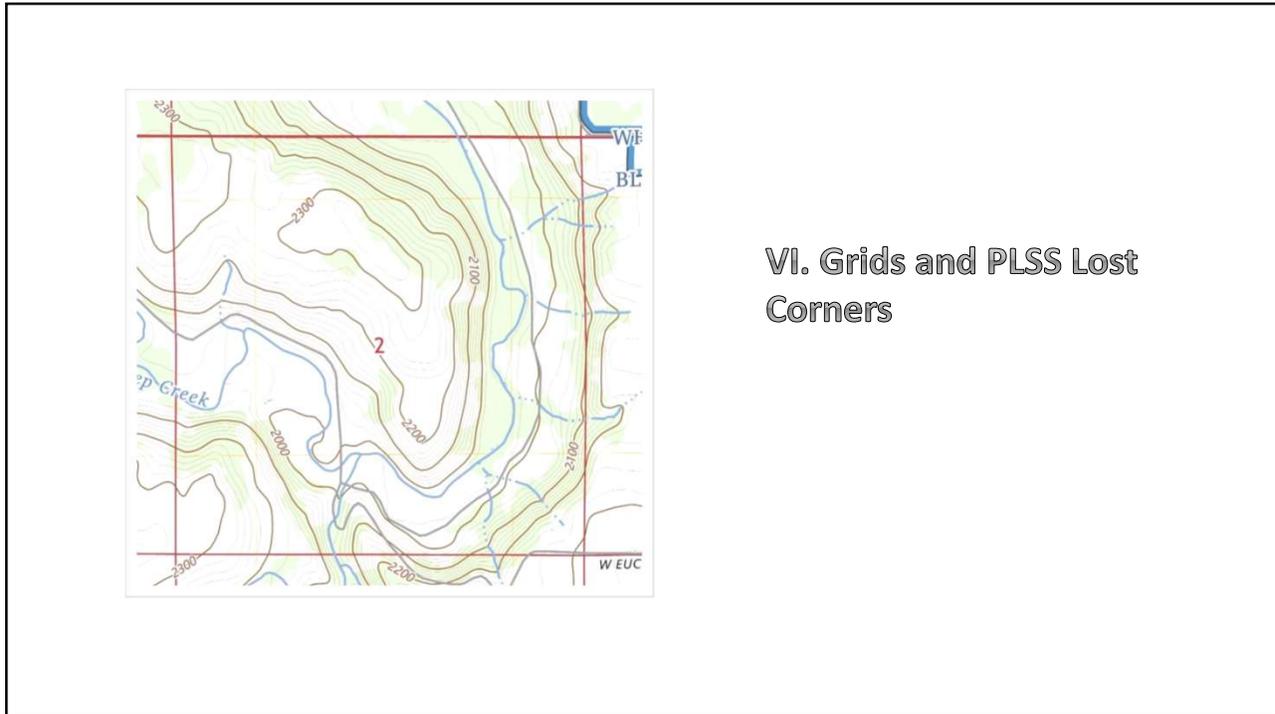
Convert to bearing

$$\text{Grid Brg} = \underline{N89^{\circ}24'10''E}$$



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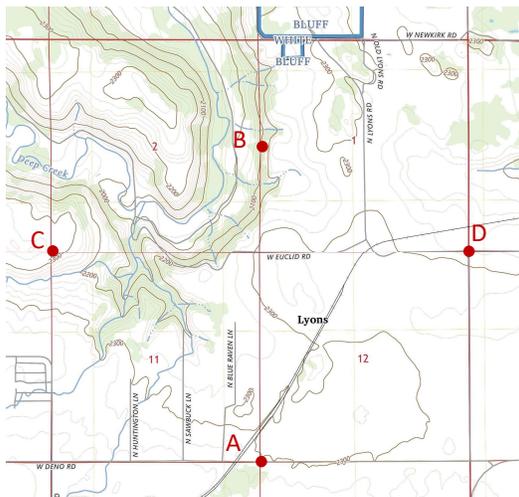


VI. Grids and PLSS Lost Corners

VI. Grids and PLSS Lost Corners



A. Cardinal Equivalents



Proportionate Measurement

Recreating original locations based on record and contemporary measurements.

Proportioning.

PLSS Manual states that proportioning must be done in *cardinal directions*

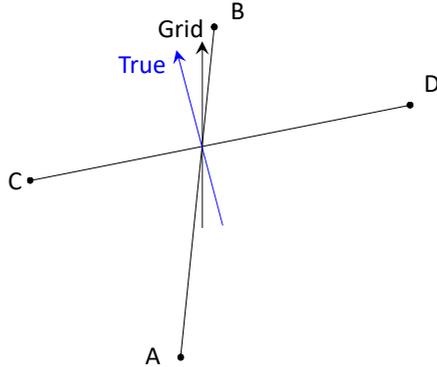
True N/S & True E/W

(*2009 Manual* treats Geodetic and True the same, which isn't technically correct, but close enough.)

VI. Grids and PLSS Lost Corners



A. Cardinal Equivalents



Proportionate Measurement

If working in a grid system must compensate for convergence.

Before or after proportioning?

How?

$\gamma = f(\text{Longitude})$ - it's not constant along E/W lines

Is its effect significant?

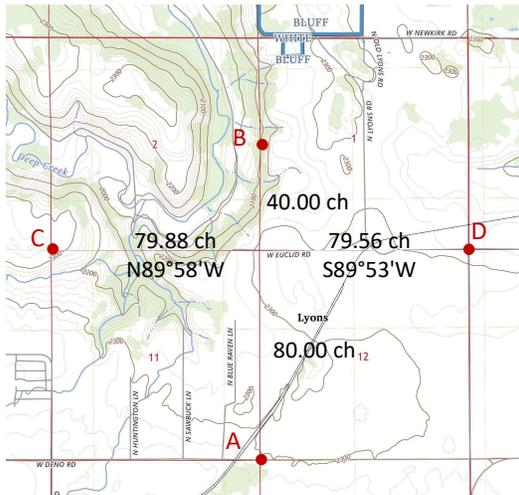
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VI. Grids and PLSS Lost Corners



A. Cardinal Equivalents



Does ignoring convergence affect corner position?

Example:

ID	North (ft)	East (ft)	k	γ
A	262,029.23	2,440,501.09	0.99997 237	2°25'13.1"
B	269,953.94	2,440,805.73	0.99996 950	2°25'13.6"
C	267,062.51	2,435,615.73	0.99997 045	2°24'15.8"
D	266,453.67	2,446,135.09	0.99997 043	2°26'10.4"

k & γ computed using NCAT

In one mile, a 2°25' rotation offset is 223 ft

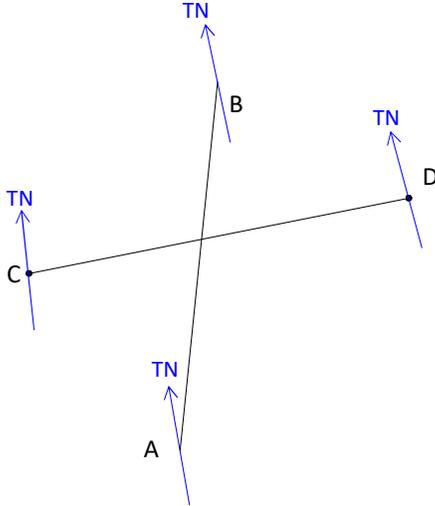
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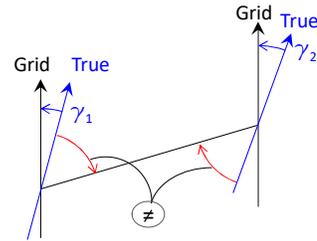
VI. Grids and PLSS Lost Corners



B. Double Proportionate Procedure



1. Compensate Before Proportioning
 Compute grid bearings from coordinates
 Determine convergence at each point
 Convert grid to true directions
 True directions are not exactly 180° apart



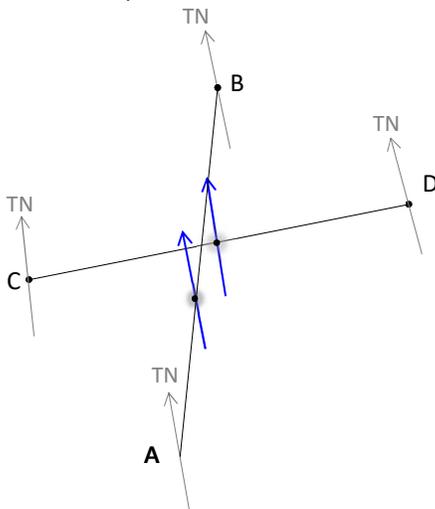
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VI. Grids and PLSS Lost Corners



B. Double Proportionate Procedure



1. Compensate Before Proportioning

PLSS Manual: use mean bearing of a line for its cardinal computations.

Compute the true bearings at line midpoints.

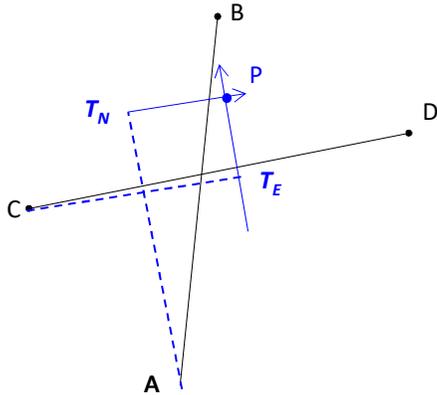
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VI. Grids and PLSS Lost Corners



B. Double Proportionate Procedure



1. Compensate Before Proportioning

PLSS Manual: use mean bearing of a line for its cardinal computations.

Compute the true bearings at line midpoints.

Then compute cardinal equivalents.

Continue regular DPM process.

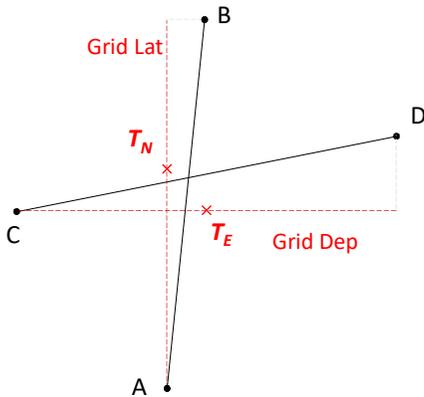
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VI. Grids and PLSS Lost Corners



B. Double Proportionate Procedure



2. Compensate After Proportioning

Set temporary points using grid equivalents

N-S	Grid Brng _{AB}	Grid Dist _{AB}
	Grid Lat _{AB}	Grid Dep _{AB}
Set T _N by SPM		
E-W	Grid Brng _{CD}	Grid Dist _{CD}
	Grid Lat _{AB}	Grid Dep _{AB}
Set T _E by SPM		

Compute γ_N and γ_E at the temp points.

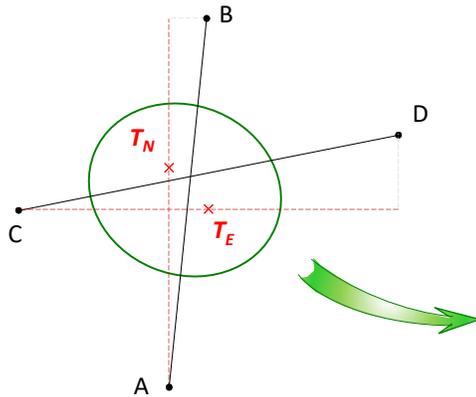
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VI. Grids and PLSS Lost Corners



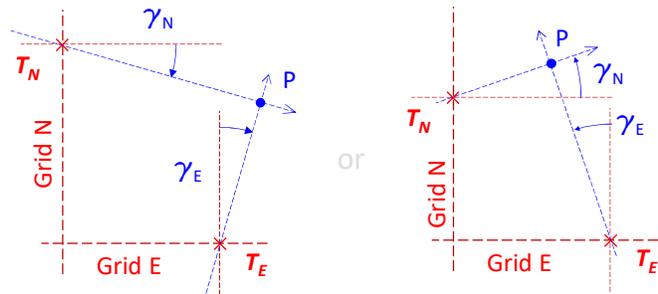
B. Double Proportionate Procedure



2. Compensate After Proportioning

At the two temporary points project lines rotated γ_N and γ_E from Grid N and Grid E.

Set lost corner at intersection of the lines.



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- I. Earth Models
- II. Spatial Systems
- III. Grid Mechanics
- IV. Formal Coordinate Systems
- V. Ground and Grid
- VI. Grids and PLSS Lost Corners

**Grid and Ground
Simple, Right?**

Learning Objectives

Mathematically modeling the Earth
Mechanics of a creating a grid system
Moving positions from 3D Earth to 2D grid
Formal Coordinate Systems
Controlling or compensating distortions
PLSS and Grid systems

Questions?



**That's It Then.
You Can Go Now.**