Measurement Errors and Their Effects

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I. Introduction

Surveyors generally perform two kinds of measurement operations:

1. Determining the spatial relationship between existing objects
2. Establishing the spatial relationship between a new and existing object

Both could be employed on any given project. Measuring between two existing property corners is an example of the first and then setting off a 10 foot building setback line would be the second.

Both operations involve making measurements, whether one-, two-, or three-dimensional. All measurements regardless the kind have one thing in common: errors. To ensure quality the surveyor must minimize errors within reason based on the application.

II. Definitions & Terms

A. Measurement

A measurement is determination of an unknown physical parameter by comparing it to a graduated device.

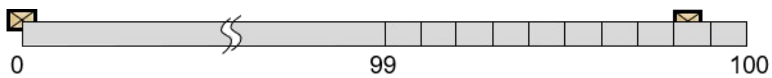


Figure 1: Taped Distance Measurement

B. Error

An error is the difference between a parameter's measurement and its true value. There are a two fundamental measurement tenets:

1. The true value of the parameter is not determinable
2. Error is always present.

Ergo: the amount of error in any measurement is never known.

Wait, we measure stuff with our really expensive equipment all the time so how can the first principle be true?

Consider the two lines in Figure 2(a): we want the angle between them.

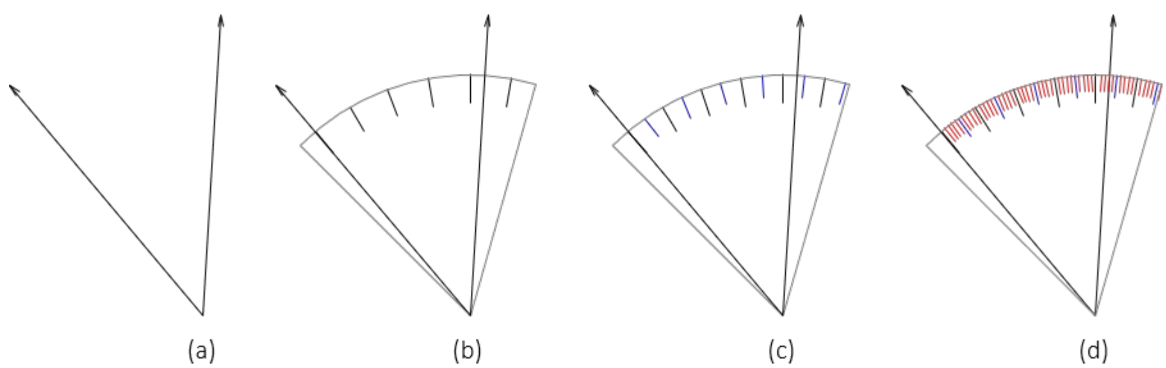


Figure 2: Angle?

In Figure 2(b) we measure the angle with an instrument whose circle is divided into 10° intervals. We can read directly to nearest 10° and interpolate 1°.

In Figure 2(c), the instrument's circle interval is 5°: read directly to nearest 5°, interpolate 1°.

In Figure 2(d), the instrument's circle interval is 1°: read directly to nearest 1°, interpolate 0.5°.

Same two lines, three different measurement values. What's the difference? Resolution. We'll get to that in a bit.

So what is the angle between the two lines? We don't know. As a consequence we don't know how much error is present. Such a dilemma.

But we do know that we can minimize the error to keep it below an acceptable level ( ).

Measurement Errors and Their Effects

C. Quality Indication

1. Precision

Precision is repeatability, how close together multiple measurements of a parameter are. We can see this because we can compare individual measurements against each other.

2. Accuracy

Accuracy is the magnitude of error in a measurement. The less error the more accurate the measurement. Since we don't know the true value of what we're measuring, we don't know how accurate our measurement is.

Comparing individual measurements to each other gives us an indication of their precision nut doesn't tell us anything about their accuracies. Precision and accuracy are independent of each other. Far any measurement set, they can have four different relationships. This is demonstrated in Figure 3 which shows four different shooters' attempts to hit a target bulls-eye with four shots.

> Shooter (a) has a tight shot group all hitting on the bulls-eye. Her shooting is precise and accurate.
>
> Shooter (b) has a tight shot group, but they're offset from the bulls-eye. His shooting is precise but not accurate.
>
> Shooter (c)'s shots are spread around on the target but their average is at about the bulls-eye. His shooting is not precise but it is accurate.
>
> Shooter (d) shouldn't be near any firearm at all. His shooting is pretty erratic and lucky to hit the target. He is neither precise nor accurate.

We want our measurements to reflect Shooter (a)'s results (although we were probably like Shooter (d) when we first started surveying).

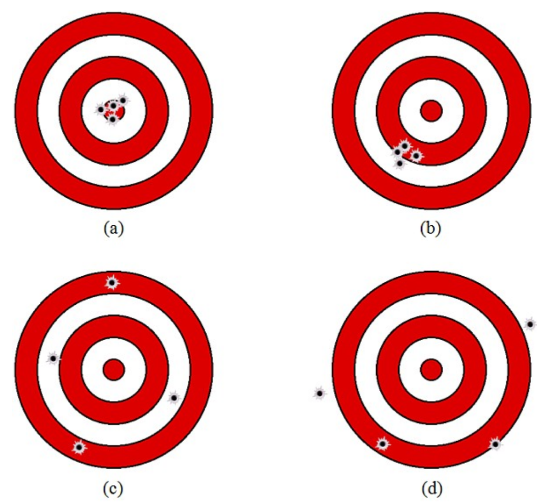


Figure 3: We Love Shootin' Stuff

3. Resolution

Recall the angle measurement example in Figure 2. The three angle measurements differed in their resolution - the smallest division on the instrument. Each successive one was a refinement of the previous one.

Does using a finer resolution make our measurements more accurate? Maybe. Theoretically it should, but we start to reach a level where creating finer divisions becomes harder to do. On a traditional transit, there's a physical limit to how small *and consistently* the divisions can be made. Total stations use electronically encoded circles which can support finer divisions, but they have their limits too. That's why it's important to be familiar with the manufacturer's stated accuracies for the different measurements supported by an instrument.

D. Error Sources

Error sources are where errors originate. There are only three sources.

Measurement Errors and Their Effects

1. Natural

Natural is the environmental within which measurements are made. It is the surrounding physical, climate, etc, conditions. Have you ever:

- sighted through an automatic level when it's very windy?
- sighted down a blacktop road on a sunny hot day?
- worked in miserable, cold, wet conditions
- been accosted by a dog? or a curious bystander?

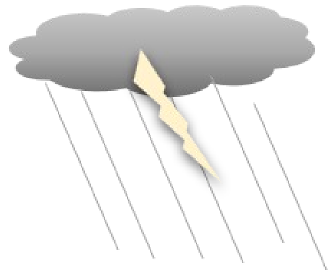


Figure 4: Into Each Life Some Rain Must Fall

Figure 5: He's Very Friendly...

2. Instrumental

These come from the limitations or maladjustment of the measuring devices.

- A bubble that runs when the instrument is rotated
- A sticking compensator in an automatic level
- Cross-hairs that don't quite look right, Figure 7

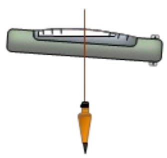


Figure 6: Bubble Run

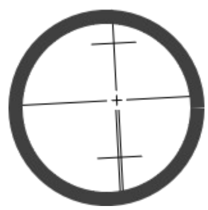


Figure 7: Cross-hairs

3. Personal

This is us, the people making the measurements. Do we know:

- how to set up equipment
- controls, menu options (so many options...)
- proper measurement procedures, including checks
- which is the appropriate equipment for the task at hand

Figure 9. Unlevel Level

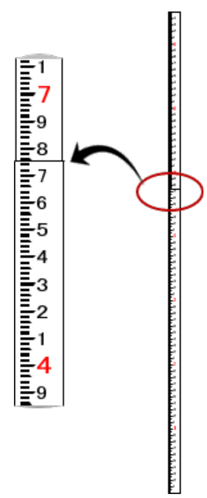


Figure 8: Not fully extended

E. Error Types

Error types define how errors behave and how they can be compensated or minimized.

1. Mistake

A mistake is a blunder, gaff, ya blew it type of thing. Common with new surveyors, us older ones will occasionally do a head-scratcher (ever set up on or measured to a wrong point?).

Usually caused by carelessness or misunderstanding, We definitely don't want to use a measurement that has a mistake in it.

2. Systematic

A systematic error conforms to some mathematical or physical law. Knowing the conditions under which equipment is used, the error is predictable. There are a few methods by which the error can be removed.

3. Random

Once we eliminate mistakes and compensate systematic errors, the only error left in a measurement is random error.

Random errors tend to be small and can be positive or negative. This means they can cancel themselves somewhat with repeated measurements.

III. Minimizing Errors

1. Mistake

You don't necessarily know a mistake has been made unless you repeat the measurement. A significant difference between the two indicates a mistake

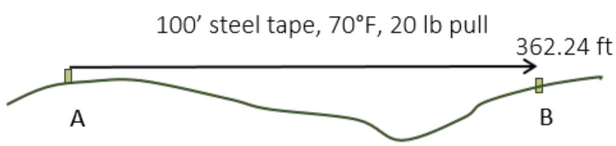


Figure 10: Distance AB

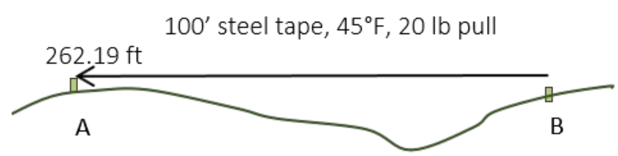


Figure 11: Distance BA

Figure 10 and Figure 11 show the results of measuring the same distance between two points in both directions using a 100 ft steel tape. The distances differ by 100 ft which indicates that either distance AB had an extra tape length accidentally included or that distance BA is missing one. We don't know which until we measure for a third time (and hopefully don't get a third wildly different length).

We eliminate mistakes by repeating the measurement and rejecting any that fall outside an acceptable range. That can be an informal range or one based on a formal standard.

2. Systematic

Because they conform to mathematical or physical law, there are a few different ways systematic errors can be compensated. These include Adjustment, Computation, and Procedure. Not all three are possible for all measurements but generally at least two are. Systematic errors can be completely eliminated.

Let's use a level's collimation error as an example.

Measurement Errors and Their Effects

A collimation error exists when the Line of Sight (LoS) is not horizontal when the instrument is correctly leveled, Figure 12. The further the rod is from the level, the larger the error's effect.

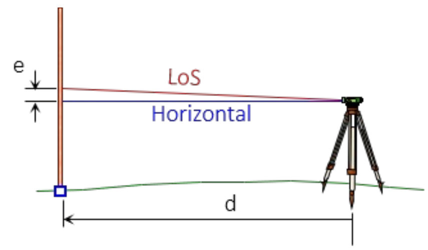


Figure 12: Collimation error

a. Adjustment

A two-peg test can performed which determines the amount the cross-hairs are too high or low. Using the vertical adjusting screws, the reticule can be moved vertically, Figure 13, until the cross-hairs are on the correct rod reading.

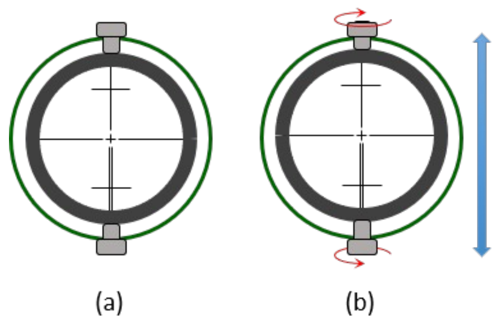


Figure 13: Reticule adjustment

b. Computation

From the peg test, the collimation error, e_1, is determined at a specific sight distance, d_1. The error is the difference between the actual and theoretical rod readings. Because the LoS is straight, the collimation correction is a linear proportion expressed as a ratio: e_1/d_1. The correction can be computed at any distance by multiplying the distance by the ratio. In Figure 14, $e_2 = d_2 \times (e_1/d_1)$. The correction is either added to or subtracted from the rod reading.

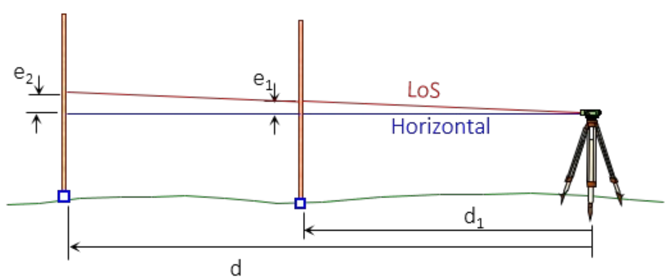


Figure 14: Collimation error behavior

c. Procedure

In differential leveling, the BS reading is added and the FS reading subtracted to determine the new elevation.

If there is a collimation issue, the BS and FS readings will both be too high or too low. If the BS and FS distances are equal, the error will be the same on both readings, Figure 15. The error on the BS is added, on the FS is subtracted so it cancels.

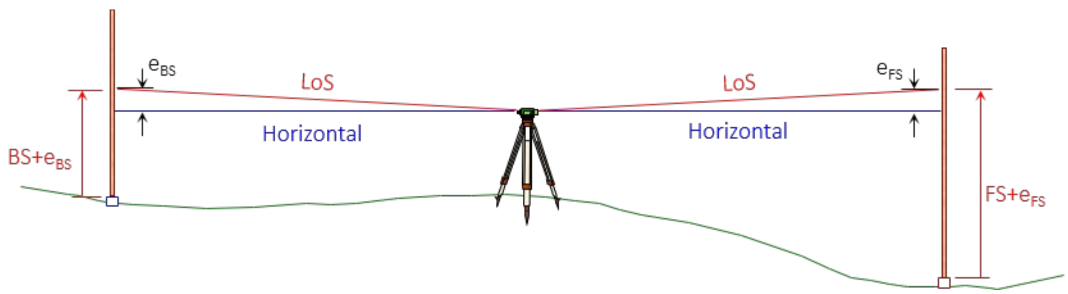


Figure 15: Procedural collimation compensation

3. Random

It's called a random error because there is no iron-clad predictability whether it will be positive or negative for a particular measurement. What we do know is that random errors tend to be small and cancel somewhat with repeated measurements.

Take the case of flipping a coin. If we flip it once, it will come up either heads or tails, we can't predict which. Flip it twice and we could get:

heads & heads
tails & tails
heads & tails
tails & heads

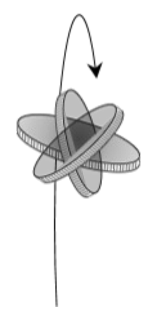


Figure 16: Coin flipping

Four combinations but half are the same. The more times we flip the coin, the closer we'll get to 50% heads and 50% tails - an even distribution of the random action. That's kind of how random errors behave. Random errors are minimized by repeating measurements sufficiently allowing them to compensate to an extent. For random errors to be completely eliminated requires an infinite number of measurements which isn't realistic. So how many times should a measurement be repeated?

For most applications where specific standards don't exist, it's up to the surveyor to decide how to proceed. There are a number of factors to consider not the least of which includes instrumentation and how quickly increased accuracy can be realized.

Figure 17 compares angle repetition measurement random errors for 3 different resolution instruments: 2", 4", and 6".

The first few repetitions for each instrument reap the greatest benefit with the largest random error compensation. After that, accuracy improvement grows at a smaller rate.

Notice also that as the number of measurements keep increasing, the accuracy discrepancy between instruments *decreases*.

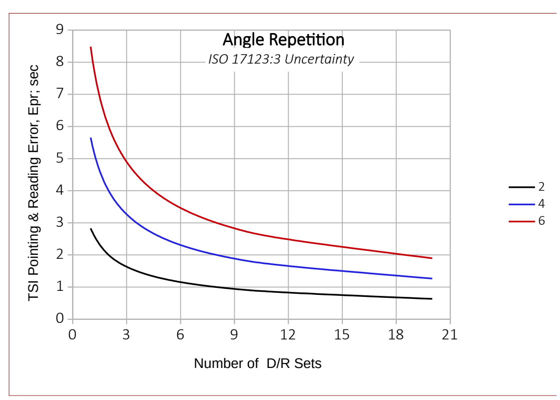


Figure 17: Angle Repetition

Measurements must as *independent* as possible to avoid repeating mistakes and to allow systematic error cancellation

IV. Random Errors

Mistakes can be eliminated and systematic errors compensated leaving only random errors affecting our measurements and results of computations in which they are used. We can use random error behavior as an accuracy indicator (remember that we never know exactly how much error is present). To do that, we need to look at some basic terms and tools

A. Terms

1. Redundancy

A redundancy, commonly called a degree of freedom, is any measurement beyond the minimum needed to determine what we're measuring.

> Example: To determine the distance between points L and K requires a single measurement. Adding a second introduces one degree of freedom; three measurements is two degrees of freedom, etc.

To determine the degrees of freedom, df:

df = m-n

m	number of measurements
n	number of unknowns

2. Discrepancy

A discrepancy is the difference between two measurements in a set of measurements.

3. Weight

A weight is a quality multiplier for an individual measurement. This allows mixing different quality measurements

4. Most probable value (MPV)

Either the simple average or the weighted average of a measurement set

simple: $MPV = \frac{\sum m_i}{n}$

weighted: $MPV = \frac{\sum (m_i \times w_i)}{\sum w_i}$

5. Residual

The difference between a measurement and the MPV:
$v = MPV - m$

6. Normal Distribution Curve

When plotted the residuals begin to form a symmetric bell-shaped curve centered on the MPV. Figure 18. The more measurements, the smoother the curve.

The curve is *asymptotic* - it approaches the x-axis bu never reaches it.

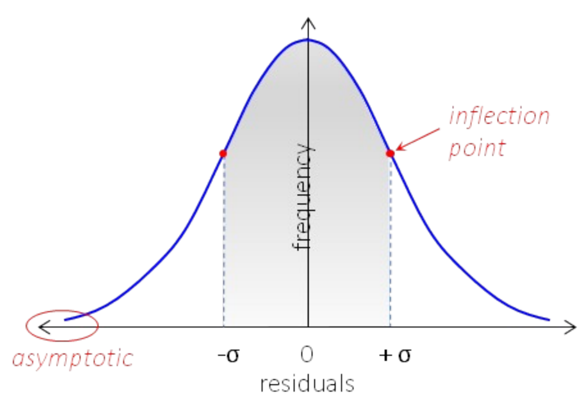


Figure 18: Normal Distribution Curve

7. Standard deviation

The standard deviation occurs the two inflection points on the normal distribution curve on each side of the MPV, Figure 18. The area under the curve between the standard deviations is approximately 68% of the total area under the curve.

$$\sigma = \sqrt{\frac{\sum(v^2)}{n-1}}$$

The smaller the standard deviation the less dispersed the measurements. Standard deviation is precision.

8. Confidence interval

A confidence interval (CI) is a range centered on the MPV that we have a certain level of confidence a measurement will fall. The confidence is the area under the curve.

The standard deviation is a ~68% CI. Other common CIs used, Figure 19, are 90%=~1.65σ and 95% =~2σ. We don't use 100% since the curve is asymptotic meaning it goes out to ±infinity.

9. Standard Error Of the Mean

The standard error of the mean, Figure 20, is an estimate of the error in the MPV, Figure 20. $E_{MPV} = \frac{\sigma}{\sqrt{n}}$

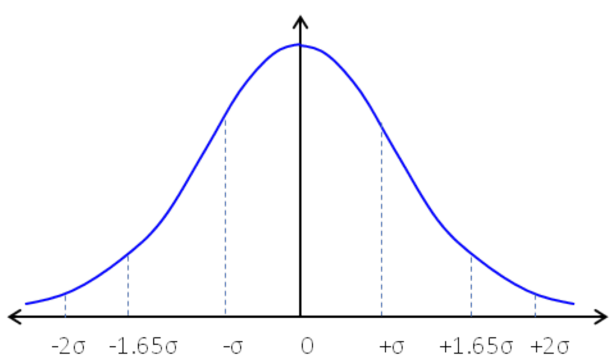


Figure 19: Confidence Intervals

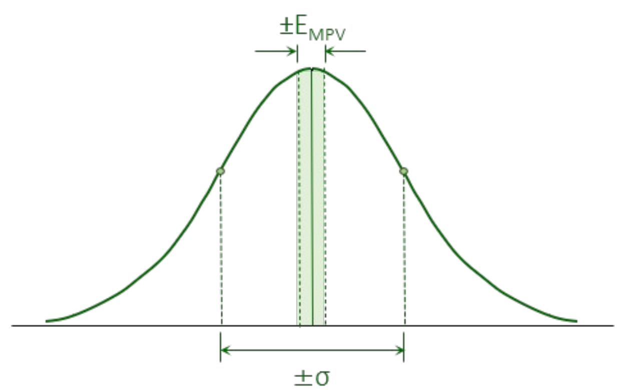


Figure 20: Error of the Mean

The standard error of the mean is an *indicator* of accuracy.

B. Error Propagation

1. Direct/Indirect Measurement

Determining the unknown we want using only an instrument is a *direct measurement*.

- To determine the height of a total station over a point a tape measure can be used.
- The atmospheric pressure needed for a total station can be read from a barometer

No other measurement or computation is necessary.

An *indirect measurement* requires a combination of measurements and math to determine something we can't measure directly.

> To determine a flagpole's height a total station is set up and the horizontal distance to the pole and vertical angle to its top are measured. Its height is computed using the measurements and trigonometry.

2. Propagation

The error in a direct measurement is determined by computing the simple or weighted average along with the standard deviation. That's it.

It's a bit more complex with an indirect measurement because other errors sources must be combined to determine their effect on the final result. It's not a simple process because of how random errors behave. This is called error propagation: individual errors are propagated into the combined result.

How errors are propagated depends on how the measurements are combined. There are many ways to combine measurements, hence many ways to propagate the errors. Some of the primary methods are discusses in this section.

a. General propagation

(1) Error of a Sum

Adding or subtracting values which are subject to errors.

$$E_{sum} = \sqrt{E_1^2 + E_2^2 + \cdots + E_n^2}$$

(2) Error of a Series

The same error occurring multiple times.

$$E_{series} = E\sqrt{n}$$

We see this one a lot in surveying. One example is allowable level loop closure.

Second Order Class II allowable loop misclosure is $C = 8\ mm\sqrt{k}$

k is loop length in km.

(3) Error of a Product

Two values with errors are multiplied or divided.

$A \pm E_A$, $B \pm E_B$

$$E_{prod} = \sqrt{\left(A \times E_B\right)^2 + \left(B \times E_A\right)^2}$$

b. Specific survey measurements

(1) Distance

$$E_{dist} = \sqrt{E_{tsi}^2 + \left[C_{msa} + \frac{D \times P_{msa}}{1,000,000}\right]^2 + E_{ref}^2}$$

Measurement Errors and Their Effects

E_{dist}	Distance error
E_{tsi}	TSI centering error
C_{msa}, P_{msa}	Manuf specified constant & ppm
E_{ref}	Reflector centering error

This is also applicable to RTK-GPS positioning.

What is the expected distance error under the following conditions?

TSI setup error: ±0.005 ft; handheld reflector centering: ±0.01 ft; 712.36 ft measured distance.

MSA: ±(3 mm + 3 ppm).

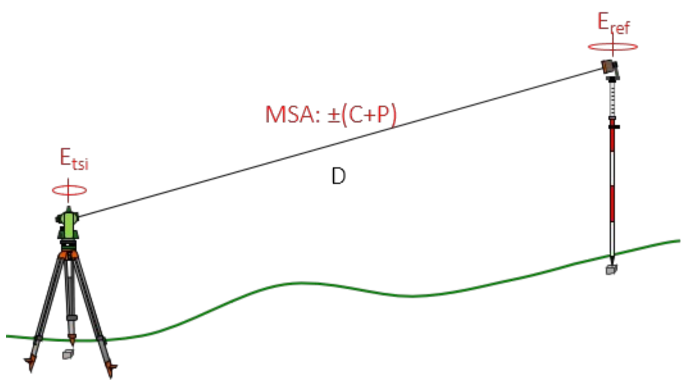


Figure 21. Distance Error

(2) TSI Horizontal Angle

Determining the expected error in a horizontal angle measurement is more complex because there are many multiple direct errors that must be propagated.

$$E_{pr} = \frac{2 \times E_{DIN}}{\sqrt{n}}$$

$$E_{tsi} = \frac{D \times E_i}{D_{BS} D_{FS} \sqrt{2}} \times \frac{206{,}264.8 \text{ sec}}{\text{radian}}$$

$$E_t = \sqrt{\left(\frac{E_{BS}}{D_{BS}}\right)^2 + \left(\frac{E_{FS}}{D_{FS}}\right)^2} \times \frac{206{,}264.8 \text{ sec}}{\text{radian}}$$

$$E_{ang} = \sqrt{E_{pr}^2 + E_{tsi}^2 + E_t^2}$$

E_{pr}	Pointing & reading error
E_{tsi}	Error due to TSI centering error
E_t	Target centering error
E_{ang}	Expected measured angle error
E_{DIN}	DIN/ISO angle accy standard
n	Number of angle measurements
D_{BS}, D_{FS}	Sight distances to BS and FS targets
E_{BS}, E_{FS}	Target centering errors of BS and FS targets.
206,264.8	Conversion from arc distance to linear.

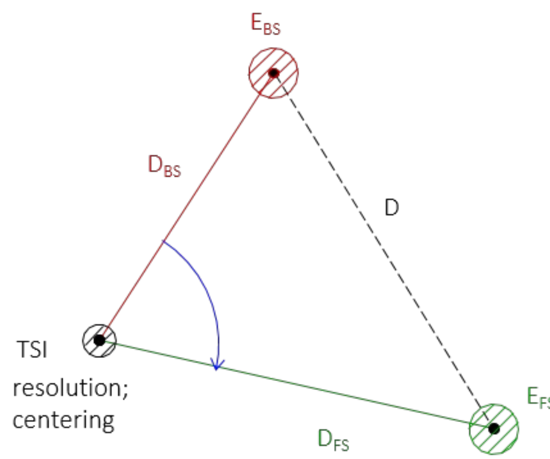


Figure 22. Horizontal Angle Error

What is the expected error in an angle measured under these conditions?

Angle: 123°30’10” meas’d 2 D/R; TSI DIN: 2 sec; TSI centering error: ±0.005 ft; BS & FS centering errors: ±0.005 ft & ±0.01 ft; BS & FS dists: 176 ft & 243 ft.

IV. Least Squares Adjustment

A. Networks and Adjustments

A network consists of a series of direct and/or indirect measurements linking together multiple unknown quantities. The network must also contain connections to known values in order to define its spatial location and orientation. A Least Squares Adjustment (LSA) minimizes the sum of the squares of the measurement residuals to determine the best position results and their uncertainties.

B. Random Errors Only

Mistakes and systematic errors must be eliminated/compensated.

A mistake's effect can be spread into other measurements; multiple mistakes can degrade the entire network.

Systematic error effects can be additive or scalar.

> An identical additive error can have different effects on different, measurements. Using a wrong reflector offset will have a larger relative impact on shorter distances tan longer ones. This uncompensated error might be found with an unconstrained adjustment.
>
> A scalar error has the same effect on all the measurements and may not be found until a fully constrained adjustment is performed. Using ground measurements instead of grid measurements may still fit together well and generally won't be revealed during an unconstrained adjustment. Once all the grid control points are included, the network will need to be distorted to fit.

For complex networks, the adjustment may be done in two steps:

1. Minimally constrained

Just enough control is included to spatially fix the network. The purpose is to see how well the measurements fit together by allowing them to move as much as possible. The effect of a mistake can be localized because only the measurements that share to point involved will be affected. This will appear as large measurement residuals (the difference between original and adjusted values).

2. Fully constrained

Measurements must be modified to fit the control network. Measurements with larger errors will change more those with smaller errors, a reflection of their quality. An adjusted network yield the best results for the unknowns, be they one-, two-, or three-dimensional positions. Ideally the measurement errors would be propagated into the final positions giving us an indication of their quality.

A fully constrained adjustment can also be used to fine-tune measurement weights.

C. Weights

Initial measurement weights are *a priori* estimates: what we think the uncertainties are before executing the LSA based on the contributing error sources. Those include things like the manufacturer's stated distance and angle accuracies, centering errors, sight lengths, etc.

Using this data, the software generates weights for the measurements. After adjustment completion a statistical analysis may be run and feedback provided on how or if to modify the weights.

Some software may limit input to manufacturers angle and distance accuracies and not allow setup errors entry.

D. Linear/Non-Linear Adjustments

An LSA minimizes the sum of the squares of the residuals of the observations: $\Sigma(v^2)$=min.

1. Vertical - Linear

Measurements are simple addition & subtraction: $Elev_B = Elev_A + BS_A - FS_B$

This is a straightforward single pass adjustment. Observation equations are created from the measurement data, matrices formed, and then solved.

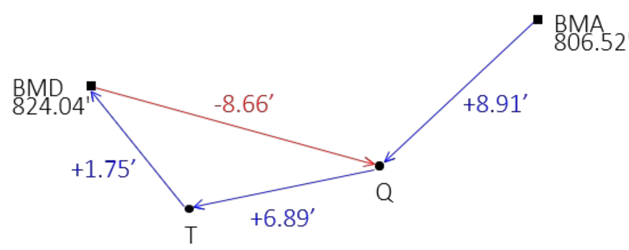


Figure 23: Level Network Adjustment

2. Horizontal - Non-Linear

Horizontal (and 3D) networks are more complex because measurements and positions are connected through trigonometric relationships which are non-linear.

An LSA solution uses coordinate variation

Starting with approximate coordinates, it uses the measurements to determine coordinate corrections.
The coordinates are updated and the process repeated.
This continues until the corrections are below an acceptable threshold

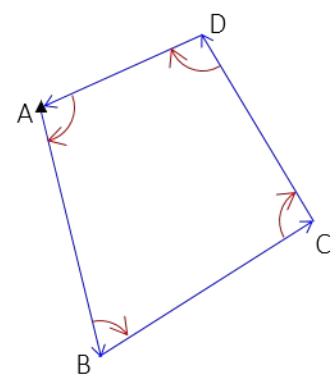


Figure 24: Horizontal

E. Adjustment Statistics

1. Overall

The Standard Deviation of Unit Weight (S_O) is a quality indicator of the overall adjustment. It is a function of the measurement residuals and degrees of freedom.

$$S_o = \sqrt{\frac{\sum_{i=1}^{m}\left(v_i^2\right)}{(m-n)}}$$

Depending on the software used, there may be other statistical quality indicators, suck as the Chi Squared Test.

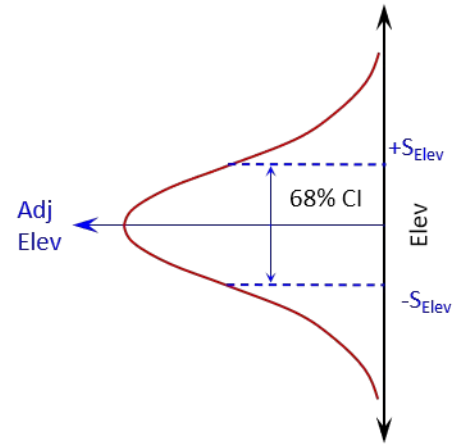
2. Unknowns

a. Vertical

Unknown elevations are determined along with their standard error,

Because leveling is a 1D adjustment the uncertainty based on a normal distribution at each elevation point, Figure 25

This is referred to as a *uni-variate* distribution because the adjusted position is a single variable.



b. Horizontal

A horizontal position consists of two related coordinates. Post-adjustment the standard deviations for both are determined, S_N & S_E, each having its own distribution Figure 26 and Figure 27.

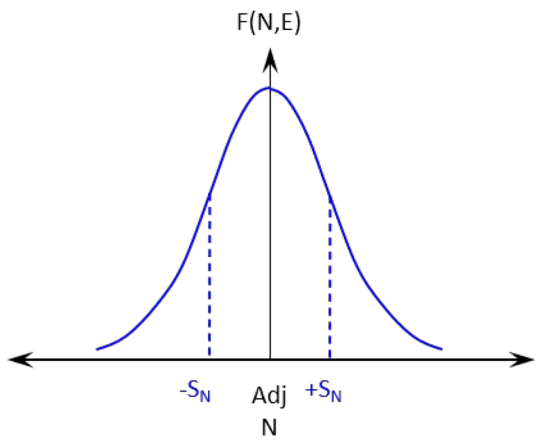


Figure 26: Adjusted N

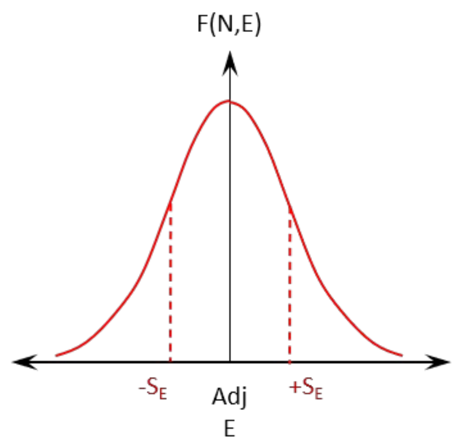


Figure 27: Adjusted E

This makes a horizontal position a *bi-variate* distribution. Instead of expressing the uncertainty individually in both directions, the distributions are combined into a 3D surface, Figure 28, and an error ellipse defined from the error triangle created from S_N and S_E, Figure 29.

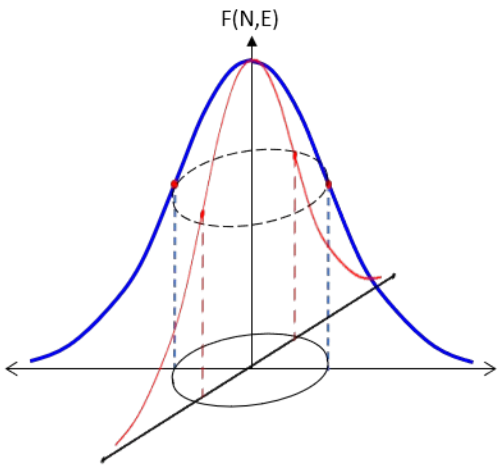


Figure 28: Bi-variate distribution

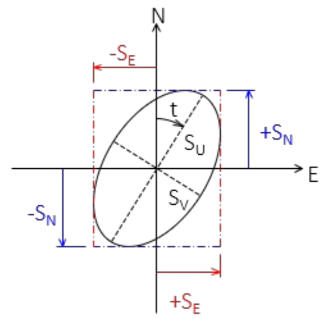


Figure 29: Standard Error Ellipse

The volume under the distribution surface bounded by the standard error ellipse is ~33-35% representing that percent confidence.

3. Horizontal Confidence Intervals

Things start to get interesting when we want to increase the CI in 2D applications because of the 3D distribution surface.

Increasing measurements increases opportunity for random errors to compensate. The more measurements, the more confident we are in the quality. The error ellipse is increased by multiplying its semi-major and -minor axes by a scalar.

Most of the time, 95% CI is used as the reporting confidence. Considering it's so universally used, figuring out the scalar should be the same, regardless the application, right?

In Chapter 19 of *Adjustment Computations: Spatial Analysis*[1] Ghilani discusses using the F-statistic to determine the CI scalar. The F-statistic is based on the desired confidence interval and the number of degrees of freedom. The scale is computed from $k=\sqrt{2\times F}$.

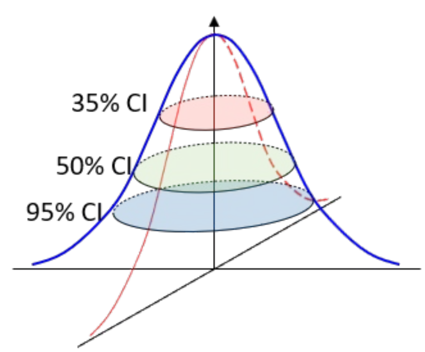


Figure 30: Different CI

The following table shows a small set of F-statistics numbers at different degrees of freedom all at the 95% CI. The corresponding scalars have been computed and added to the table:

DF	F	k	DF	F	k
1	199.5	20.0	4	6.94	3.72
2	19.0	6.76	5	5.79	3.40
3	9.55	4.37	10	4.10	2.86

[1]Fifth edition, 2010, John Wiley & Sons, Inc, Hoboken NJ

Note that the scalar at one degree of freedom is 4.5 times that at two and 7 times that at 20 degrees of freedom. The greater the degrees of freedom, the smaller the 95% CI - we're more certain with more measurements.

OK, so that's one way to determine the scalar.

ALTA/NSPS Land Title Survey Standards defines the Relative Positional Precision at the "95 percent level, or approximately 2 standard deviations." That's smaller than the F-statistic scalar even at 10 degrees of freedom. Why 2? Recall that the 95% CI for a uni-variate distribution is approximately 1.96 times the standard deviation, commonly rounded to 2. But doubling the error ellipse axes doesn't create a 95% volume under the distribution surface.

Most software includes *ALTA/NSPS* standards verification, but they aren't consistent in how the 95% CI error ellipse is determined.

Traverse PC and *SALSA* use the F-statistic

Trimble software and *StarNet* both use a ~2.45 multiplier regardless the degrees of freedom

(2.45 is the 95% F-statistic for an infinite number of degrees of freedom)

Not all software allows a priori errors entry. Most will accept manufacturer's instrument specifications but some don't have provision for equipment centering errors.

Does any of this make a difference? Well...

4. Pin cushions

Have you ever encountered a pin cushion (aka, monument nest), Figure 31? Contributed to one?

Figure 31: Pin cushion examples

If measurement uncertainties are taken into account, there may be no statistical difference in the positions. Adding one more monument may just add to the confusion.

F. Example

1. Adjacent Parcel Surveys

Two adjacent parcels are described by metes and bounds (neither calls the other as an adjoiner), Figure 32. They share a boundary and the beginning point for each description is the common southerly corner (labeled A and J).

All the monuments are found except for the common northerly one which is lost (D/M).

Anderson's property was surveyed in 2021 by Jones using a simple loop traverse. She held direction of line AB and the record distance and direction of line C to D.

Wright's property was surveyed in 2023 by Mills also using a simple loop traverse. He held the record direction of line JK and record distance and direction of line LM.

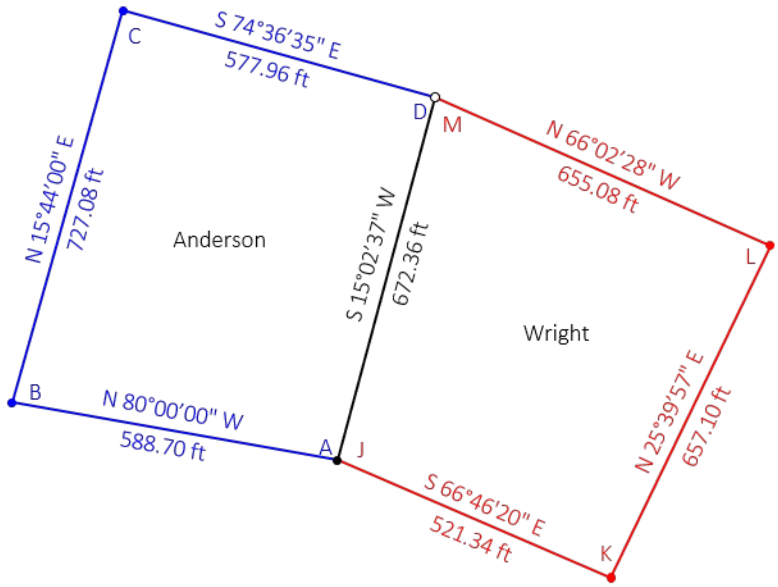


Figure 32: Record dimensions

2. Adjustment Results & Analysis

Both surveys were combined and adjusted in *StarNet* using a 95% CI.

The adjusted coordinates and standard deviations of the common corner at are:

Parcel	*North, ft*	*East, ft*	S_N	S_E
Anderson	2649.294	1174.077	0.138	0.221
Wright	2649.267	1174.507	0.121	0.209

The two positions are separated by just under half a foot, Figure 33.

StarNet uses 2.4477, regardless the number of degrees of freedom, to scale the standard error ellipse to 95% CI.

There were 9 df in the combined adjustment. The 95% CI F-statistic for 9 is 6.9443, the scalar is 2.918.

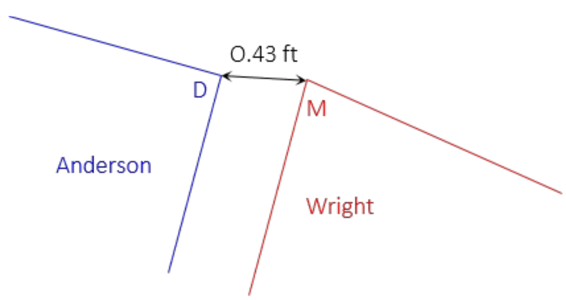


Figure 33: Common corner separation

The next table shows the lengths of semi-major and -minor axes for the standard, *StarNet* 95% CI, and F-statistic error ellipses for the two positions. The semi-major axis azimuth is also shown so the ellipses can be oriented correctly.

	Standard			*StarNet 95%; k=2.4477*		*F-statistic 95%, k=2.918*	
	a	*b*	*Aza*	*a*	*b*	*a*	*b*
Anderson	0.225	0.130	104°19’	0.552	0.319	0.658	0.380
Wright	0.217	0.106	100°00’	0.531	0.260	0.633	0.310

The *StarNet* and F-statistic 95% CI error ellipses are plotted to scale in Figure 34 and Figure 35.

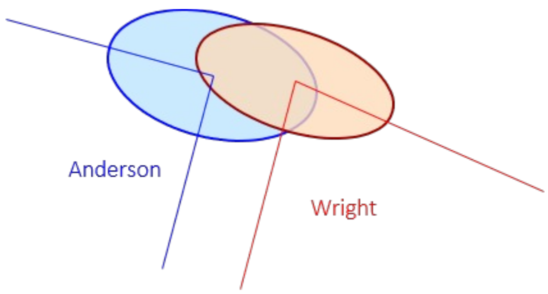


Figure 34: StarNet 95% CI Ellipses

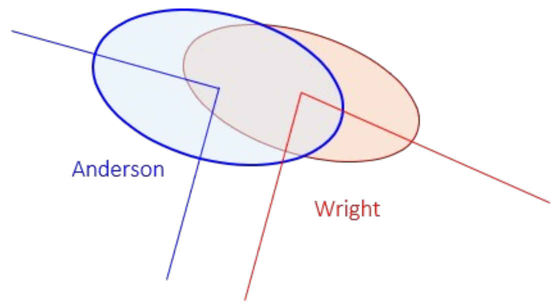


Figure 35: F-statistic 95% CI Ellipses

Notice that each point position is located inside the error ellipse of the other. Using a 95% confidence interval. statistically there’s no difference between the positions.

If there are no monuments already set (and no senior rights involved) split the difference and set one there.

If there are already monuments in place, pick one that’s closest to the middle and run with it. There’s no reason to set another one based on the position uncertainty, Figure 36.

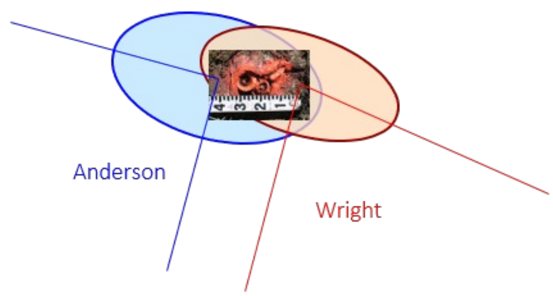


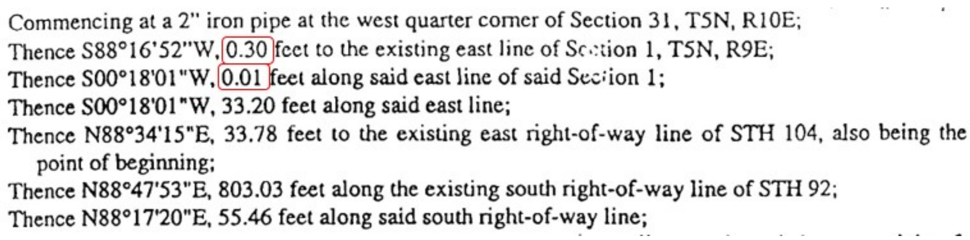
Figure 36: Multiple monuments

The only reasons there are multiple monuments at a single location:

- Different evidence was used or wasn’t applied correctly
- No account for measurements error effects
- Both

3. But wait, there’s more....

If it isn’t bad enough that we have monument nests, we sometimes documenting similar issues in the public record. The partial description in Figure 37 was taken directly from a recorded deed.



Commencing at a 2" iron pipe at the west quarter corner of Section 31, T5N, R10E;
Thence S88°16'52"W, 0.30 feet to the existing east line of Section 1, T5N, R9E;
Thence S00°18'01"W, 0.01 feet along said east line of said Section 1;
Thence S00°18'01"W, 33.20 feet along said east line;
Thence N88°34'15"E, 33.78 feet to the existing east right-of-way line of STH 104, also being the point of beginning;
Thence N88°47'53"E, 803.03 feet along the existing south right-of-way line of STH 92;
Thence N88°17'20"E, 55.46 feet along said south right-of-way line;

Figure 37: Now that's accuracy

Go ahead: measure 0.30 ft, turn 87°58'51", then measure 0.01 ft. Go on, I dare you.

G. Summary

1. A priori uncertainties

We mentioned the ALTA/NSPS Land Title Surveys standards earlier. Section 3.e.i. defines the Relative Positional Precision at the 95% CI and says it can "be estimated by the results of a correctly weighted least squares adjustment of the survey." Correctly weighting means appropriate a priori uncertainties assigned and refined based on the adjustment results. Not all adjustment software supports complete a apriori uncertainties entry.

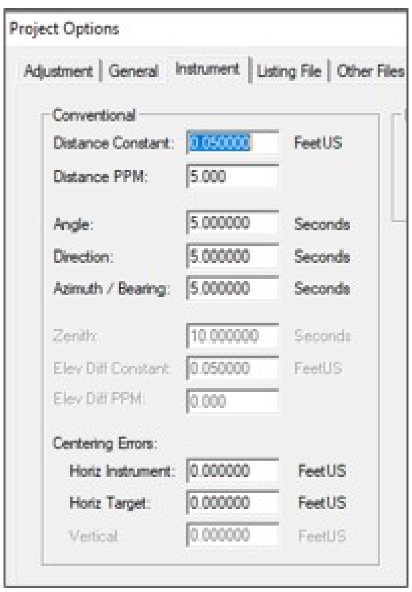


Figure 38: StarNet a pirori entry

Figure 39: Traverse PC a pirori entry

StarNet, Figure 38, supports entry of instrumental and personal uncertainties. *Traverse PC*, Figure 39, allows only instrumental uncertainties. Control coordinate uncertainties are supported by both programs. Traverse PC uses the same errors for all the control points in a project; StarNet allows individual control coordinate error entry directly in the data file(s).

Which a priori values are needed for a *correctly weighted least squares adjustment of the survey*? Well ... all of them.

2. Who’s in charge?

We know that minimizing errors requires correct use of equipment - that also applies to software. We may not like it, but if required to we’re able to defend our measurements, evidence evaluation, and decisions thereof in court. What about our software’s analysis? We are as responsible for knowledgeable use of software as for instruments.

For your adjustment software you *must* know:

- How your standard error ellipses are scaled to 95% CI
- Which a priori uncertainties you are able to enter

If these aren’t apparent from the input options or adjustment results, then (shudder) check the software documentation.

And when you next encounter a pin cushion, before popping in another monument squint your eyes and see if you can make out the error ellipse.