Mentoring Mondays - 5 Feb 2024

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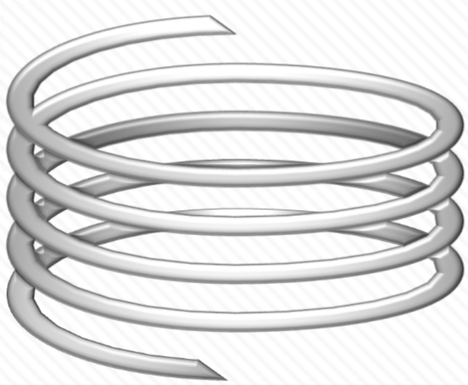

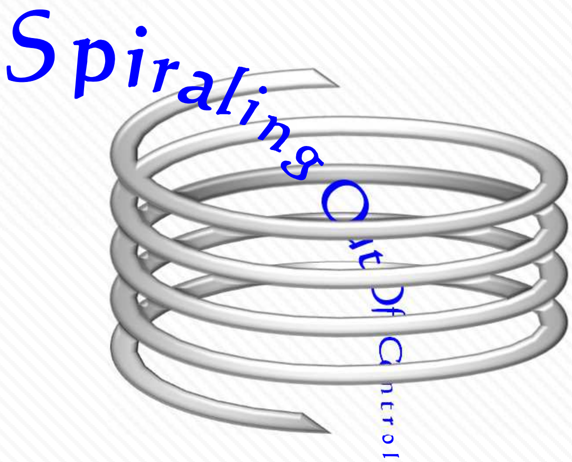

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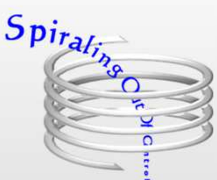


OUTLINE

I. Background
II. Spirals: What all There Is
III. Spiral by Deflection Angles
IV. Examples
V. Spreadsheet

I. BACKGROUND

A. Introduction

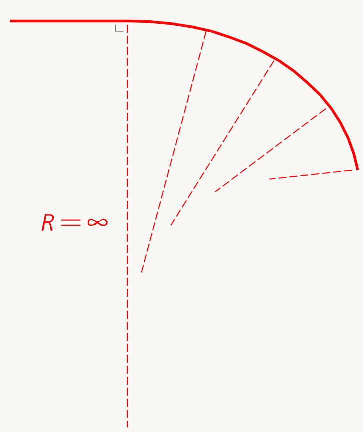
B. Basic Vehicle Dynamics

C. Spiraled Curves

I. A. Introduction

Spiral

- Curve with uniformly changing radius
  - More complex than circular arc.
  - Complex math can be simplified somewhat.



I. B. Basic Vehicle Dynamics

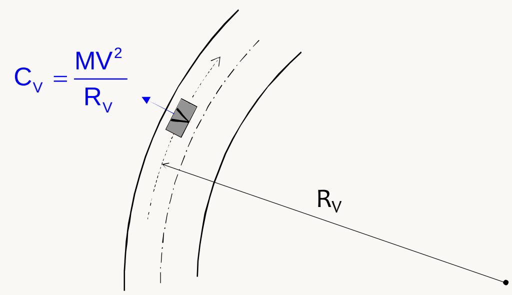
1. Centrifugal Force

An object traveling a curved path is subject to an outward force, C_V

C_V =F(Velocity, Radius)

⇧V, ⇧C_V

⇩R, ⇧C_V



I. B. Basic Vehicle Dynamics

2. Superelevation, e

Pavement inclined to inside of curve.

Part of vehicle mass used to offset C_V.

Tire friction & e counteract C_V.

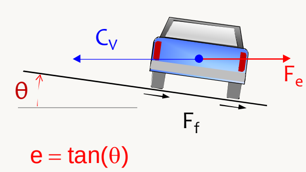
Speed range of curve.

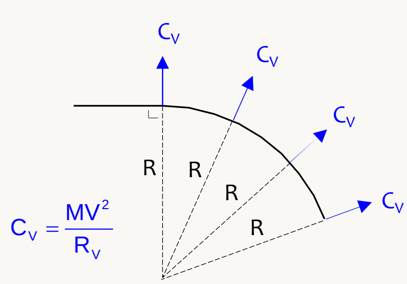
On tangent: R=∞ so C_V=0

Circular arc: Full C_V at beginning?

Driver can compensate

Flatter curves





I. B. Basic Vehicle Dynamics

2. Superelevation, e

A curve has a speed range

Max V = e + F_f

Min V = e – F_f

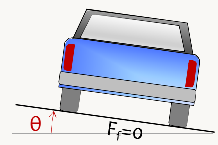
Equilibrium ("hands-off") speed

Vehicle forces balanced.

Won't slide in or out on frictionless pavement.

Vehicle "drives" itself around the curve.

$$V_E = \sqrt{\frac{C_v R_v}{M}}$$



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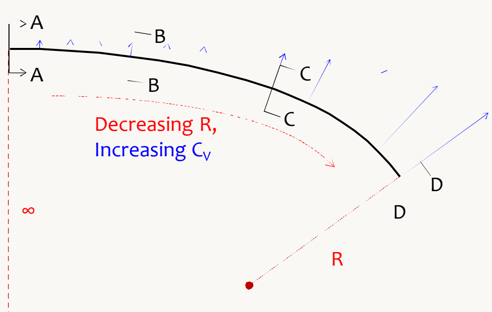
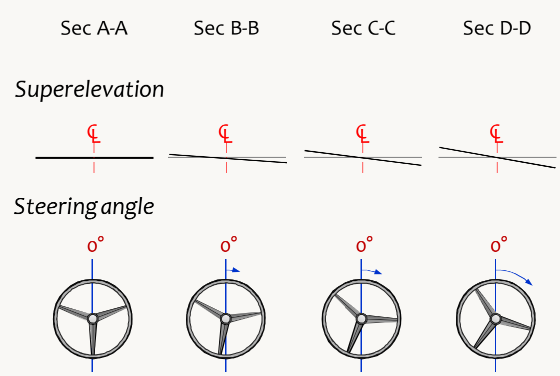
I. B. Basic Vehicle Dynamics

2. Superelevation, e

Spiral: Radius decreases introducing C_{IV}.

Allows gradual e and turning angle introduction.





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I. B. Basic Vehicle Dynamics

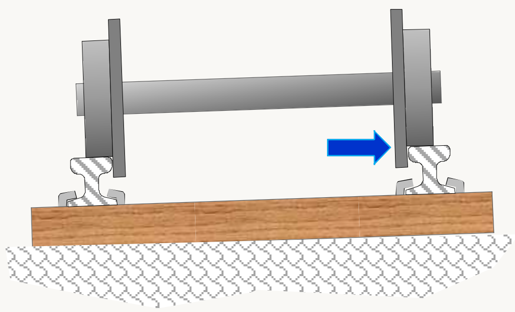
3. Trains

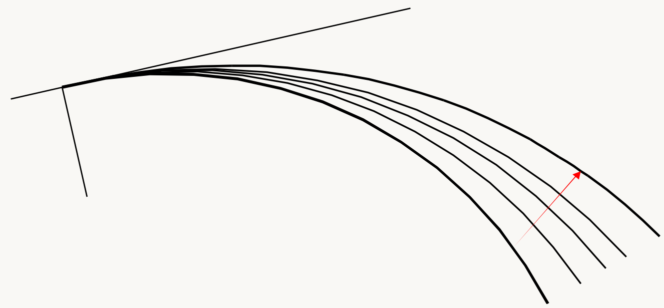
Mechanical connection between train and rails.

Train transmits C_V directly to rails.

Outer rail raised to introduce superelevation

Track laid out as a circular curve will gradually shift into a spiral configuration.





I. C. Spiraled Curves

1. Transition Spirals

Entrance into identical Exit

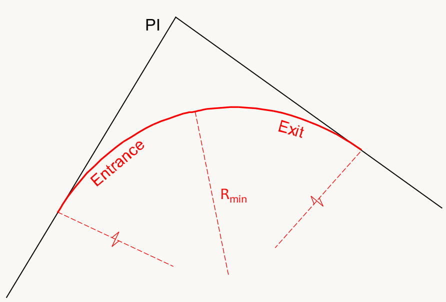
Smooth continuous transition throughout.

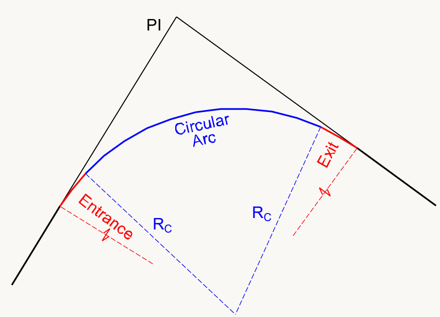
More complicated to compute.

Generally use short spirals on both sides of circular arc.

Long enough to introduce and remove superelevation.

Simpler to compute





I. C. Spiraled Curves

1. Transition Spirals

70mph				
R	L	X	T	e (%)
3,330	225	90	315	5.0%
3,220	230	90	320	5.1%
3,120	234	90	324	5.2%
3,010	239	90	329	5.3%
2,910	243	90	333	5.4%
2,800	248	90	338	5.5%
2,700	252	90	342	5.6%
2,580	257	90	347	5.7%
2,460	261	90	351	5.8%
2,310	266	90	356	5.9%
2,040	270	90	360	6.0%

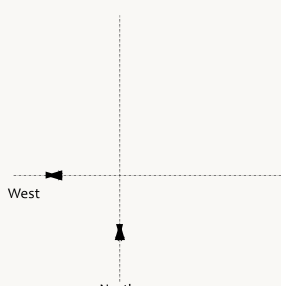
WisDOT *Facilities Design Manual*

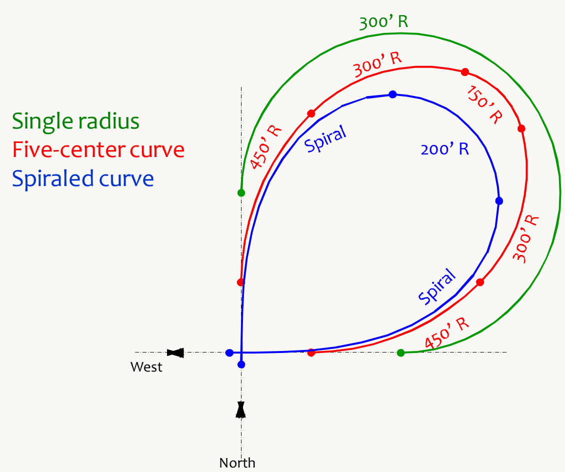
Max Design Velocity: 70 mph

- ⇩R, ⇧e
- Full e = 6.0% at min R = 2040 ft
- Transition distance L = 270 ft

I. C. Spiraled Curves

2. Cloverleaf





II. SPIRALS: WHAT ALL THERE IS

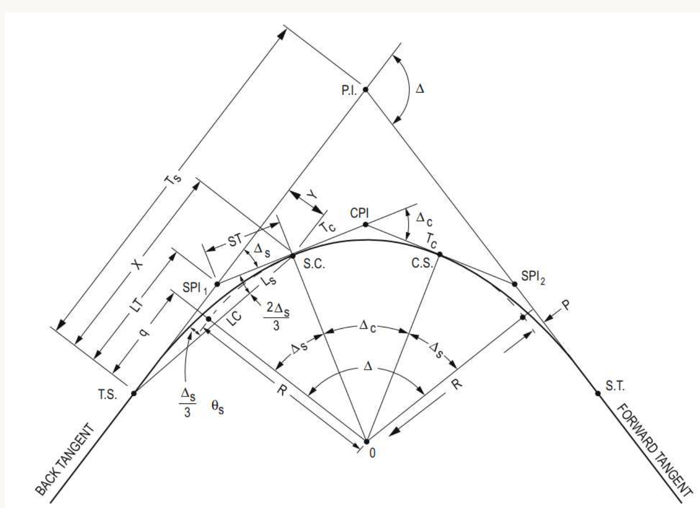
A. Diagram

B. Constructing a Spiraled Curve

C. Nomenclature; Parts

II. A. Diagram

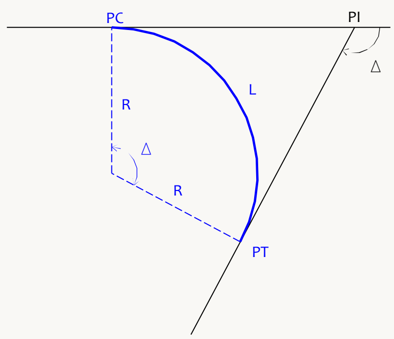
NCEES *Principles and Practice of Surveying Reference Handbook* Edition 1.2



II. B. Inserting Spirals

Circular curve

Curve transitions thru Δ degrees.

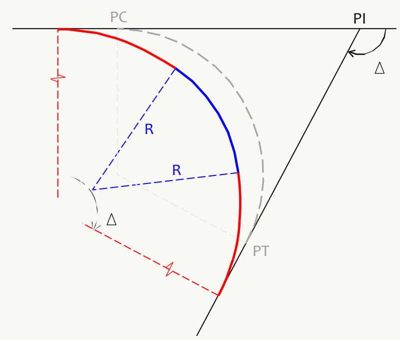


Adding spirals

Pushes curve system further from PI

Shortens length of circular arc.

Total Δ is same as for the single curve.



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II. C. Nomenclature; Parts

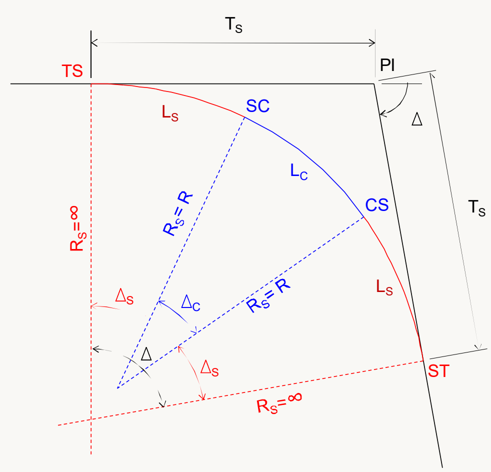
1. Spiraled Curve

   Usual defining elements

   - Δ
   - R or D for circular arc
   - Spiral length, L_S

   Curve endpoints

   - TS: Tangent to Spiral
   - SC: Spiral to Curve
   - CS: Curve to Spiral
   - ST: Spiral to Tangent



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II. C. Nomenclature; Parts

1. Spiraled Curve

Central angles

Δ_S: Entrance & Exit spiral

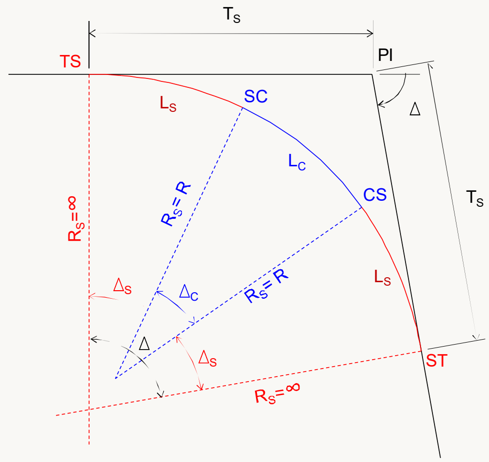
Δ_C: Circular arc

$\Delta = \Delta_C + 2\Delta_S$

$\Delta_S = (L_S\Delta)/200$

$\Delta_C = \Delta - 2\Delta_S$

.



II. C. Nomenclature; Parts

1. Spiraled Curve

Circular curve components

Computed using regular equations except Δ_C used instead of Δ.

Length

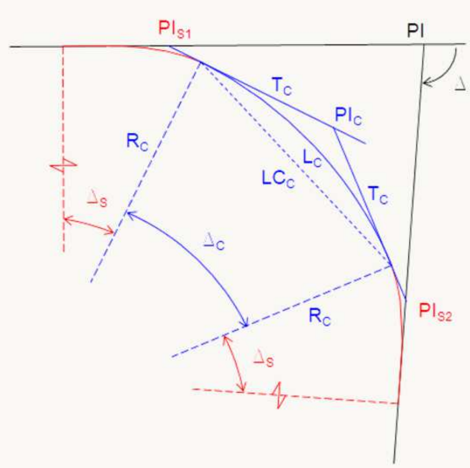
$$L_c = \frac{100\Delta_c}{D}$$

Long chord

$$LC_c = 2R\sin\left(\Delta_c/2\right)$$

Tangent

$$T_c = R\tan\left(\Delta_c/2\right)$$



II. C. Nomenclature; Parts

2. Stationing

Most complicated part of spiral comps is computing T_S.

- A: Spiral defl angle at flat end,
- X & Y: tangent dist & offset to SC.
- X_o & o: tangent dist & offset to OPC

$$A \approx \frac{\Delta_S}{3}$$

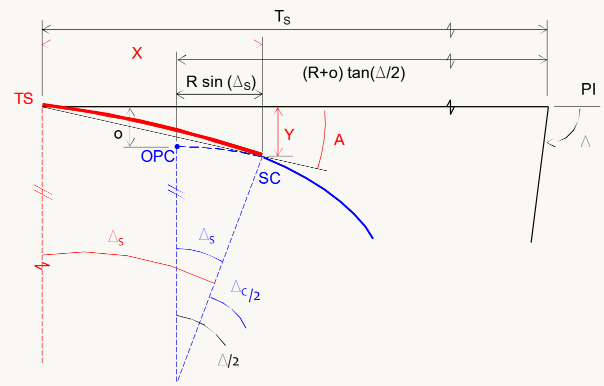
$$Y \approx L_S \sin\left(\frac{\Delta_S}{3}\right)$$

$$X \approx L_S - \left(\frac{Y^2}{2L_S}\right)$$

$$X_O = X - R_c \sin \Delta_S$$

$$o = Y - R_c\,(1 - \cos \Delta_S)$$

$$T_S = X - R_c \sin(\Delta_S) + (R_c + o)\tan\left(\Delta/2\right)$$



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II. C. Nomenclature; Parts

2. Stationing

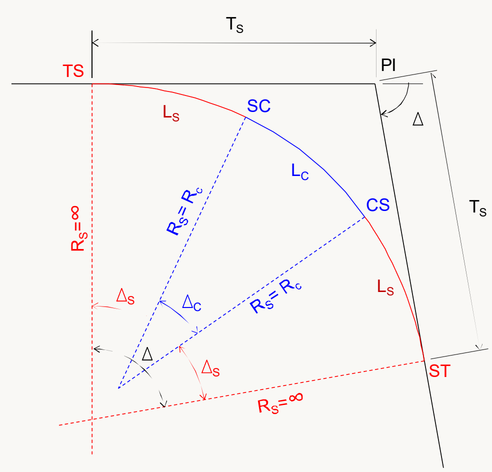
$$Sta_{TS} = Sta_{PI} - T_S$$

$$Sta_{SC} = Sta_{TS} + L_S$$

$$Sta_{CS} = Sta_{SC} + L_C$$

$$Sta_{ST} = Sta_{CS} + L_S \quad \textit{Back}$$

$$Sta_{ST} = Sta_{PI} + T_S \quad \textit{Ahead}$$



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III. SPIRAL BY DEFLECTION ANGLES

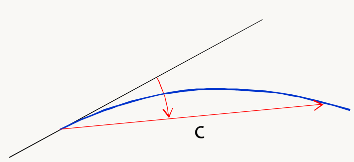
A. Approximate Method
B. Tangent Offset Method
C. Power Series Method
D. Spiral Interval
E. Circular Arc
F. Staking the Spiraled Curve

III. Spiral by Deflection Angles

Similar to circular curve deflection angle method.

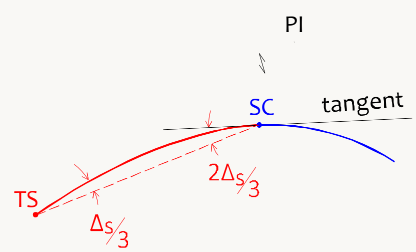
From start of curve:

- Turn deflection angle
- Measure radial chord



Spiral is not a symmetric curve

- defl angle at flat end = $\Delta_S/3$
- defl angle at sharp end = $(2\Delta_S)/3$



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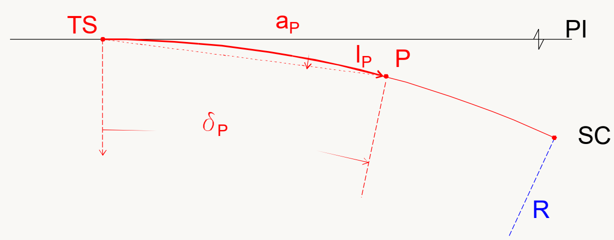
III. A. Approximate Method

This method uses arc distances as chord distances.

The deflection angle is proportional to the square of the distance along the spiral.

$$l_P = Sta_P - Sta_{TS}$$

$$a_P = \left[\frac{l_p}{L_S}\right]^2 \left[\frac{\Delta_S}{3}\right]$$



III. B. Tangent Offset Method

Curve points located by:

Tangent distance, x

Tangent offset, y

Deflection angle and chord are computed from x & y

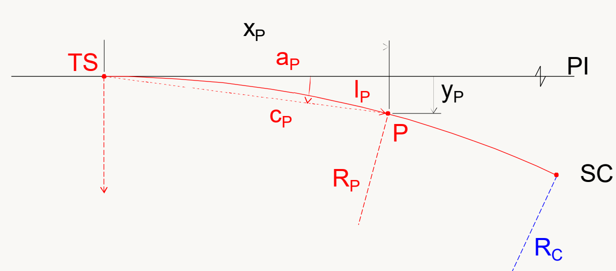
$$R_p = R_c \left[\frac{L_S}{l_p}\right]$$

$$y_P = \frac{l_p^3}{6R_p L_S}$$

$$x_p = l_p - \frac{y_p^2}{2l_p}$$

$$a_p = \tan^{-1}\left(\frac{y_p}{x_p}\right)$$

$$c_p = \sqrt{x_p^2 + y_p^2}$$



III. C. Power Series Method

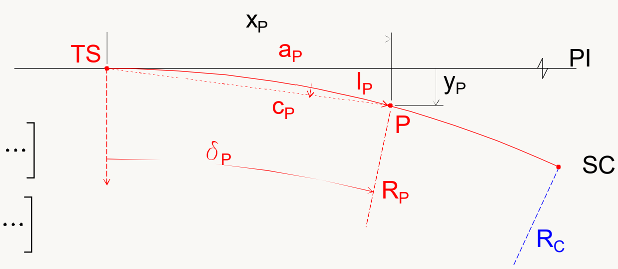
A more exact version of the Tangent Offset Method.

- x and y computed using infinite series power equations.
  - δ_P is spiral central angle in radians for the curve point.

$$\delta_P = \frac{l_P^2}{2R_P L_S}$$

$$x_p = l_p\left[1 - \frac{\delta_p^2}{5(2!)} + \frac{\delta_p^4}{9(4!)} - \frac{\delta_p^6}{13(6!)} + \cdots\right]$$

$$y_P = l_p\left[\frac{\delta}{3} - \frac{\delta_p^3}{7(3!)} + \frac{\delta_p^5}{11(5!)} - \frac{\delta_p^7}{15(7!)} \cdots\right]$$



Both are infinite series but first four terms are sufficient.

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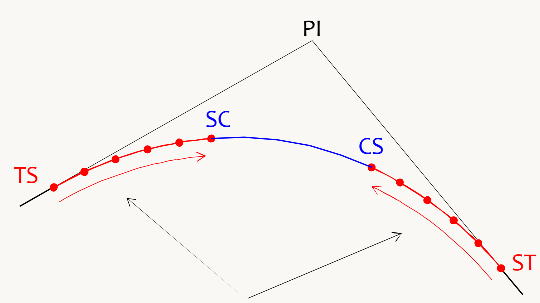
III. D. Spiral Interval

Spirals are short

Entrance & exit identical but reversed

To simplify comps

- Compute in equal intervals
- Commonly 5- or 10-chord
- Can use same comps for both spirals



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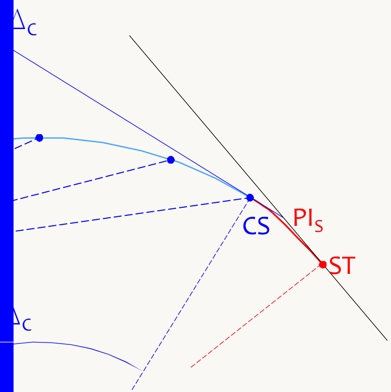


III. F. Traditional Stakeout

1. Stake TS & ST

    Measure T_S from PI along tangents.

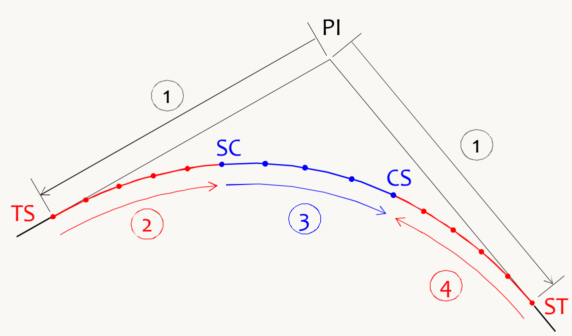
2. Stake entrance spiral

    Set up on TS

    BS PI

    Measure deflection angle and chord to set curve point

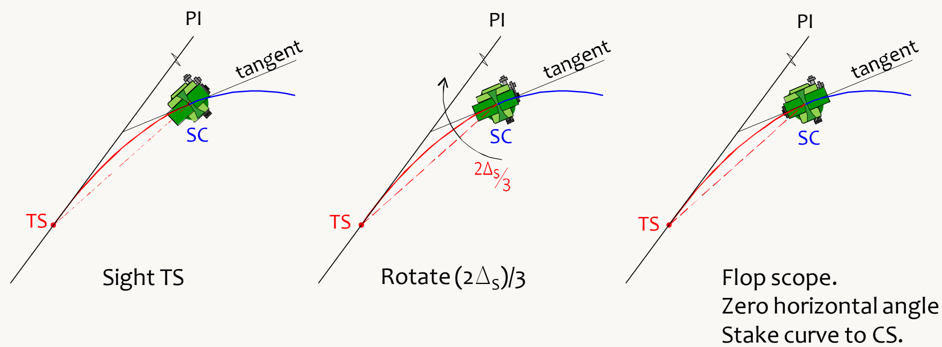
    Continue to SC.



III. F. Traditional Stakeout

3. Stake circular curve

Set up on SC and orient to the circular arc.

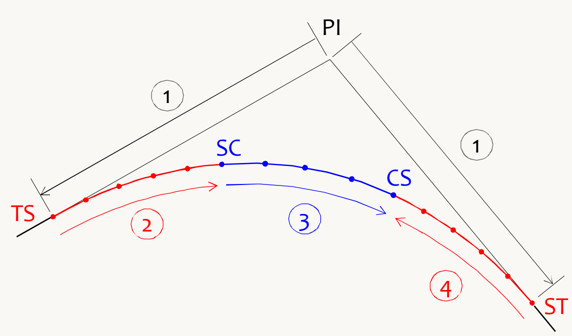


III. F. Traditional Stakeout

4. Stake exit spiral

Same comps and procedure as entrance spiral,

Start at ST, close on already staked CS



IV. EXAMPLES

A. Example 1
B. Example 2
C. Example 3

IV. A. Example 1

450.00 ft long entrance and exit spirals will be used with a 3000.00 radius circular curve. The PI station is 52+00 with a Δ angle of 47°00' Left.

Part (1) What is the total deflection angle of the entrance spiral at the TS?

Part (2) What is length of the circular curve?

Part (3) What is the deflection angle at the TS to the spiral midpoint?

Solution

IV. B. Example 2

A 300.00 ft long spiral will be used with a 3°00' circular curve. The PI station is 40+00 and the Δ angle is 60°00' Right. Using the Approximate Method, compute the five-chord deflection angle notes for the spirals.

Solution

IV. C. Example 3

A 275.00 ft long spiral will be used with a 2500.00 ft radius circular curve. The PI station is 63+00 and the Δ angle is 38°00' Left. Determine the curve system endpoint stations.

Solution

SOLUTIONS

IV. A. Example 1

450.00 ft long entrance and exit spirals will be used with a 3000.00 radius circular curve. The PI station is 52+00 with a Δ angle of 47°00' Left.

Part (1) What is the total deflection angle of the entrance spiral at the TS?

$$D = \frac{5729.58}{3000.00} = 1°54'35.5''$$

$$\Delta_S = \frac{L_S D}{200} = \frac{(450.00)\,(1°54'35.5'')}{200} = 4°17'50'$$

$$A = \frac{\Delta_S}{3} = \frac{4°17'50''}{3} = 1°25'57''$$

IV. A. Example 1

450.00 ft long entrance and exit spirals will be used with a 3000.00 radius circular curve. The PI station is 52+00 with a Δ angle of 47º00' Left.

Part (2) What is length of the circular curve?

$$\Delta_C = \Delta - 2\Delta_C = 47°00' - 2(4°17'50'') = 38°24'20''$$

$$L_C = \frac{100\Delta_C}{D} = \frac{100(38°24'20'')}{1°54'35.5''} = 2010.90\ ft$$

IV. A. Example 1

450.00 ft long entrance and exit spirals will be used with a 3000.00 radius circular curve. The PI station is 52+00 with a Δ angle of 47º00' Left.

Part (3) What is the deflection angle at the TS to the spiral midpoint?

Distance to the midpoint is 450.00/2 = 225.00 ft

$$a_P = \left[\frac{l_P}{L_S}\right]^2 \left[\frac{\Delta_S}{3}\right] = \left[\frac{225.00}{450.00}\right]^2 \left[\frac{4°17'50''}{3}\right] = 0°21'29''$$

Example 2

IV. B. Example 2

A 300.00 ft long spiral will be used with a 3°00' circular curve. The PI station is 40+00 and the Δ angle is 60°00' Right. Using the Approximate Method, compute the five-chord deflection angle notes for the spirals.

Central Angles

$$\Delta_S = \frac{L_S D}{200} = \frac{(300.00)\,(3°00')}{200} = 4°30'$$

$$\Delta_C = \Delta - 2\Delta_C = 60°00' - 2(4°30') = 51°00'$$

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IV. B. Example 2

A 300.00 ft long spiral will be used with a 3°00' circular curve. The PI station is 40+00 and the Δ angle is 60°00' Right. Using the Approximate Method, compute the five-chord deflection angle notes for the spirals.

Spiral deflection angle comps

$$Subchord = {}^{300.00}\!/_{5} = 60.00\ ft \qquad a_P = \left[\frac{l_P}{L_S}\right]^2 \left[\frac{\Delta_S}{3}\right] = \left[\frac{l_P}{300.00}\right]^2 \left[\frac{4°30'}{3}\right]$$

Chord #	l_p, ft	a_p
1		
2		
3		
4		
5		

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IV. B. Example 2

A 300.00 ft long spiral will be used with a 3°00' circular curve. The PI station is 40+00 and the Δ angle is 60°00' Right. Using the Approximate Method, compute the five-chord deflection angle notes for the spirals.

Spiral deflection angle comps

$$Subchord = 300.00 \big/ 5 = 60.00\ ft \qquad a_P = \left[\frac{l_P}{L_S}\right]^2 \left[\frac{\Delta_S}{3}\right] = \left[\frac{l_P}{300.00}\right]^2 \left[\frac{4°30'}{3}\right]$$

Chord #	l_p, ft	a_p
1	60.00	0°03’36”
2	120.00	0°14’24”
3	180.00	0°32’24”
4	240.00	0°57’36”
5	300.00	1°30’00”

Example 3

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IV. C. Example 3

A 275.00 ft long spiral will be used with a 2500.00 ft radius circular curve. The PI station is 63+00 and the Δ angle is 38°00' Left. Determine the curve system endpoint stations.

Central Angles

$$D = \frac{5729.58}{2500.00} = 2°17'30.6''$$

$$\Delta_S = \frac{L_S D}{200} = \frac{(275.00)\ (2°17'30.6'')}{200} = 3°09'04.6''$$

$$\Delta_C = \Delta - 2\Delta_C = 38°00' - 2(3°09'04.6'') = 31°41'50.8''$$

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IV. C. Example 3

A 275.00 ft long spiral will be used with a 2500.00 ft radius circular curve. The PI station is 63+00 and the Δ angle is 38º00' Left. Determine the curve system endpoint stations.

Central Angles

$$D=\frac{5729.58}{2500.00}=2°17'30.6''$$

$$\Delta_S=\frac{L_S D}{200}=\frac{(275.00)\,(2°17'30.6'')}{200}=3°09'04.6''$$

$$\Delta_C=\Delta-2\Delta_C=38°00'-2(3°09'04.6'')=31°41'50.8''$$

Curve Length

$$L_C=\frac{100\Delta_C}{D}=\frac{100\,(31°41'50.8'')}{2°17'30.6''}=1383.06\ ft$$

IV. C. Example 3

A 275.00 ft long spiral will be used with a 2500.00 ft radius circular curve. The PI station is 63+00 and the Δ angle is 38º00' Left. Determine the curve system endpoint stations.

Spiraled Tangent

$$T_S=X_O+(R+o)\tan\left(\Delta/2\right)$$

$$Y\approx L_S\sin\left(\frac{\Delta_S}{3}\right)=275.00\times\sin\left(\frac{3°09'04.6''}{3}\right)=5.04\ ft$$

$$X\approx L_s-\left(\frac{Y^2}{2L_S}\right)=275.00-\left(\frac{5.04^2}{2\times275.00}\right)=274.95\ ft$$

$$X_O=X-R\sin(\Delta_S)=274.95-2500.00\times\sin(3°09'04.6'')=137.52\ ft$$

$$o=Y-R(1-\cos\Delta_S)=5.04-2500.00\,(1-\cos(3°09'04.6''))=1.26\ ft$$

$$T_S=X_O+(R+o)\tan\left(\Delta/2\right)=137.52+(2500.00+1.26)\tan\left(38°00'/2\right)=998.77\ ft$$

IV. C. Example 3

A 275.00 ft long spiral will be used with a 2500.00 ft radius circular curve. The PI station is 63+00 and the Δ angle is 38°00' Left. Determine the curve system endpoint stations.

Stationing

$$Sta_{TS} = Sta_{PI} - T_S = (63+00) - 998.77 = 53+01.23$$
$$Sta_{SC} = Sta_{TS} + L_S = (53+01.23) + 275.00 = 55+76.23$$
$$Sta_{CS} = Sta_{SC} + L_C = (55+76.23) + 1383.06 = 69+59.29$$
$$Sta_{ST} = Sta_{CS} + L_S = (69+59.29) + 275.00 = 72+34.29 \ \textit{Back}$$
$$Sta_{ST} = Sta_{PI} + T_S = (63+00) + 998.78 = 72+98.78 \ \textit{Ahead}$$

V. SPREADSHEET

V. Spreadsheet

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A	B	C	D	E	F	G	H	I	J	K
Spiral Computations, v1.2			Instructions							16-Dec-23
PI Station	63+00.00				**Random comp**					
Total Δ	38.00000	DD.MMSSs			Spiral arc	180.00				
R for Circular Arc	2500.00				Defl ang	+0°27'00.1"				
Spiral Length, L_s	275.00				Chord	180.00				
Spiral Components				**Arc Components**					**Stationing**	
Δ change rate, k	0.0083	deg/linear unit			Deg Curve	2.29183	Dec deg		Spiral Tangent, T_s	998.78
Spiral Angle, Δ_s	3.1513	Dec Deg				+2°17'30.6"	DMS		TS	53+01.22
Spiral Angle, Δ_s	+3°09'04.6"				Central angle, Δ_c	31.6975	Dec deg		SC	55+76.22
Spiral Defl angle, A	+1°03'01.5"					+31°41'50.9"	DMS		CS	69+59.29
Tan dist to end, X	274.95	Approx Method			Arc length, L_c	1383.06			ST Bk	72+34.29
	274.92	Power Series			Long chord, LC_c	1365.49			ST Ah	72+98.78
Tan offset to end, Y	5.04	Approx Method			Tangent, T_c	709.73				
	5.04	Power Series			Tan dist to OPC, x_o	137.52				
					Tan offset to OPC, o	1.26				

V. Spreadsheet

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Five-chord Spiral															
Chord Length	55.00														
	Entrance	Exit	*Approx method*			*Tangent Offset Method*				*Power series*					
Num	Sta	Sta	Arc dist, l_i	Radius, R_i	Defl ang, a_i	Dist, x_i	Offset, y_i	Chord, c_i	Defl ang, a_i	δ_i, radians	Tan dist, x_i	Tan offset, y_i	Chord, c_i	Defl ang, a_i	
1	53+56.22	69+59.29	55.00	12,500.00	+0°02'31.3"	55.00	0.04	55.00	+0°02'31.3"	0.002200	55.00	0.04	55.00	+0°02'31.3"	
2	54+11.22	70+14.29	110.00	6,250.00	+0°10'05.0"	110.00	0.32	110.00	+0°10'05.0"	0.008800	110.00	0.32	110.00	+0°10'05.0"	
3	54+66.22	70+69.29	165.00	4,166.67	+0°22'41.3"	165.00	1.09	165.00	+0°22'41.4"	0.019800	164.99	1.09	165.00	+0°22'41.3"	
4	55+21.22	71+24.29	220.00	3,125.00	+0°40'20.2"	219.98	2.58	220.00	+0°40'20.2"	0.035200	219.97	2.58	219.99	+0°40'20.1"	
5	55+76.22	71+79.29	275.00	2,500.00	+1°03'01.5"	274.95	5.04	275.00	+1°03'01.7"	0.055000	274.92	5.04	274.96	+1°03'01.4"	

Ten-chord Spiral														
Chord Length	27.50													
	Entrance	Exit	*Approx method*							*Power series*				
Num	Sta	Sta	Arc dist, l_i	Radius, r_i	Defl ang, a_i	Dist, x_i	Offset, y_i	Chord, c_i	Defl ang, a_i	δ_i, radians	Tan dist, x_i	Tan offset, y_i	Chord, c_i	Defl ang, a_i
1	53+28.72	69+59.29	27.50	25,000.00	+0°00'37.8"	27.50	0.01	27.50	+0°00'37.8"	0.000550	27.50	0.01	27.50	+0°00'37.8"
2	53+56.22	69+86.79	55.00	12,500.00	+0°02'31.3"	55.00	0.04	55.00	+0°02'31.3"	0.002200	55.00	0.04	55.00	+0°02'31.3"
3	53+83.72	70+14.29	82.50	8,333.33	+0°05'40.3"	82.50	0.14	82.50	+0°05'40.3"	0.004950	82.50	0.14	82.50	+0°05'40.3"
4	54+11.22	70+41.79	110.00	6,250.00	+0°10'05.0"	110.00	0.32	110.00	+0°10'05.0"	0.008800	110.00	0.32	110.00	+0°10'05.0"
5	54+38.72	70+69.29	137.50	5,000.00	+0°15'45.4"	137.50	0.63	137.50	+0°15'45.4"	0.013750	137.50	0.63	137.50	+0°15'45.4"
6	54+66.22	70+96.79	165.00	4,166.67	+0°22'41.3"	165.00	1.09	165.00	+0°22'41.4"	0.019800	164.99	1.09	165.00	+0°22'41.3"
7	54+93.72	71+24.29	192.50	3,571.43	+0°30'52.9"	192.49	1.73	192.50	+0°30'53.0"	0.026950	192.49	1.73	192.49	+0°30'52.9"
8	55+21.22	71+51.79	220.00	3,125.00	+0°40'20.2"	219.98	2.58	220.00	+0°40'20.2"	0.035200	219.97	2.58	219.99	+0°40'20.1"
9	55+48.72	71+79.29	247.50	2,777.78	+0°51'03.0"	247.47	3.68	247.50	+0°51'03.1"	0.044550	247.45	3.67	247.48	+0°51'03.0"
10	55+76.22	72+06.79	275.00	2,500.00	+1°03'01.5"	274.95	5.04	275.00	+1°03'01.7"	0.055000	274.92	5.04	274.96	+1°03'01.4"

QUESTIONS?

Mentoring Mondays - 5 Feb 2024

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