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I. Background

A. Introduction

A spiral is a particular kind of horizontal curve whose radius decreases or increases along its entire length, Figure 1.

A complete spiral is more complex than a circular arc and more difficult to compute. So if they're more complicated, why use them?

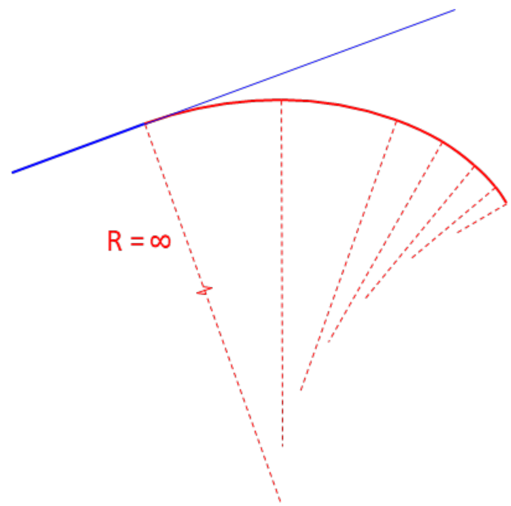


Figure 1

B. Basic Vehicle Dynamics

1. Centrifugal Force

When a vehicle travels in a straight direction, it does not experience any lateral force. As it travels around a curve while maintaining a constant velocity, Figure 2, the vehicle experiences a force, C_V, pushing it outward. Decreasing the curve radius and/or increasing the velocity increases the force pushing out.

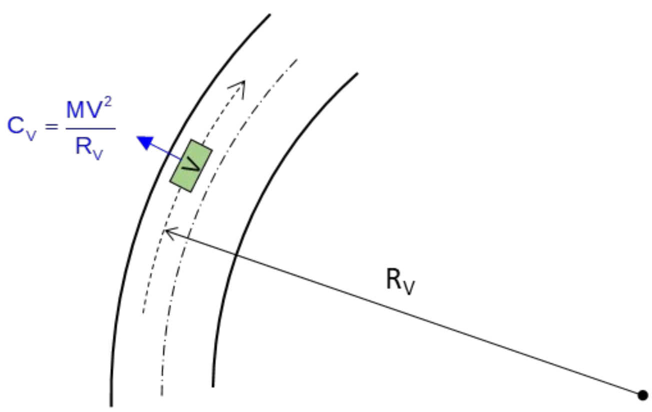


Figure 2

$$C_v = \frac{MV^2}{R_V}$$

2. Superelevation

To help offset the centrifugal force acting on a vehicle as it traverses a curve, the road surface is inclined toward the inside of the curve, Figure 3. This is called superelevation. As the centrifugal force increases the pavement slope increases.

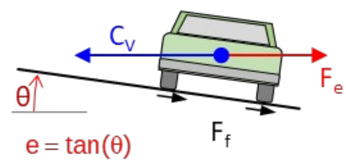


Figure 3

The combination of superelevation and tire-pavement friction create a speed range within which a vehicle can travel without sliding off the curve. At the right speed a vehicle can go around a frictionless (think: ice-covered) surface without sliding off, Figure 4; this is the curve's *equilibrium speed.* Because forces are balanced theoretically the vehicle can "drive" itself around the curve with no driver input.[1]

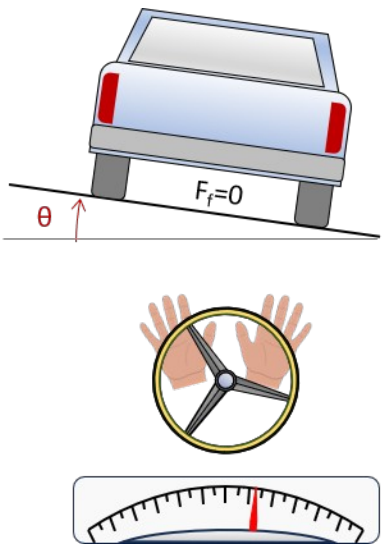


Figure 4

In theory, as soon as a vehicle enters a constant radius curve, Figure 5, it experiences the full centrifugal force. In practice, this is not the case because the driver has sufficient space within a lane to react and steer into the curve. Using very flat curves in high speed situations makes the transition from no- to full-centrifugal force less abrupt.

A spiral provides a convenient way to introduce superelevation as the centrifugal force on the vehicle increases going into the curve, Figure 6.

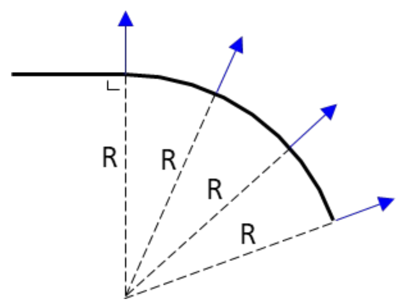


Figure 5

1 The equilibrium speed is also referred to as the "hands-off speed". Experimentation to discover a particular curve's hands-off speed is **not** encouraged.

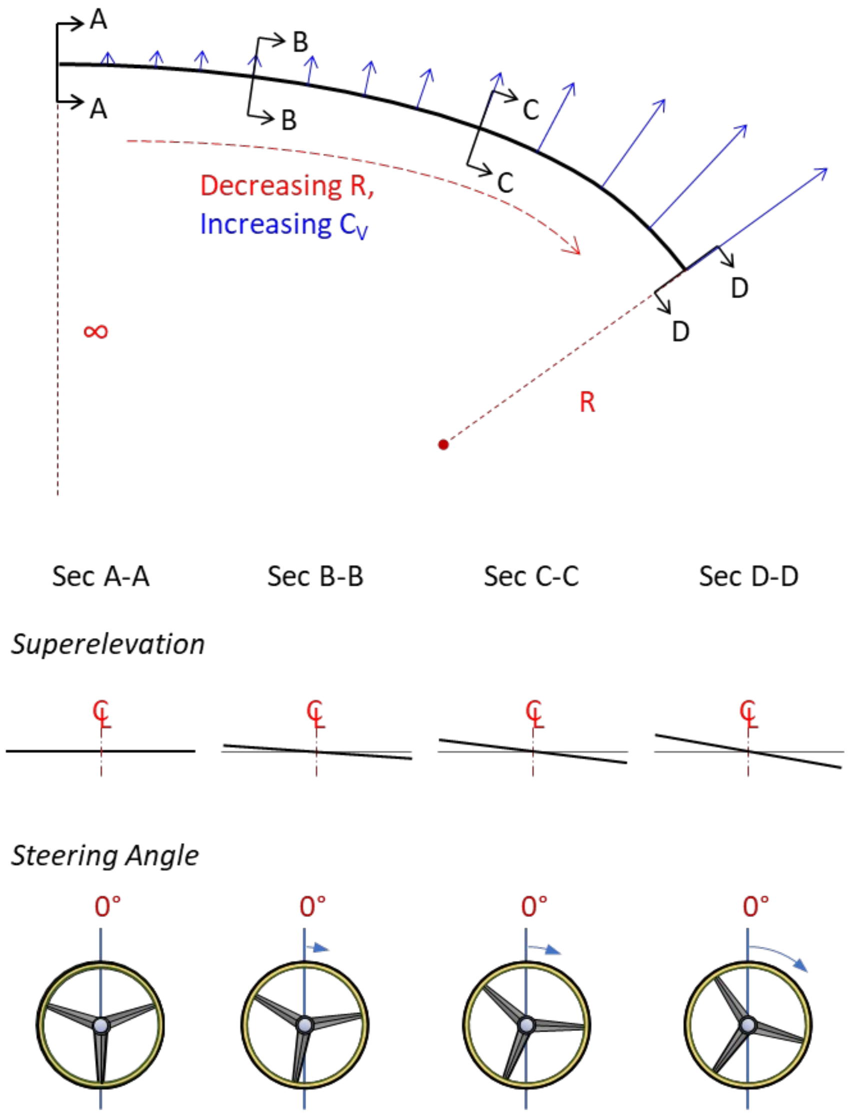


Figure 6

At the beginning of the spiral where there is no centrifugal force acting on the vehicle (Sec A-A), superelevation is not necessary. Moving through the spiral the superelevation increases to offset the increasing centrifugal force as the radius decreases; it acts to balance vehicle forces. Full superelevation and centrifugal force are achieved at the end of the spiral (Sec D-D).

Instead of an abrupt steering adjustment at the beginning of a circular curve, the driver gradually turns the wheel in a continuous fashion through the spiral curve.

3. Trains

Train dynamics differ from a car: the engineer does not steer so is not able to adjust the travel path entering a fixed-radius curve. The train changes direction based on its direct contact with the rails. The outside rail is higher than the inside one introducing superelevation, Figure 7. As the train enters the curve the tracks must offset the centrifugal force acting on the train. Running trains repeatedly through a fixed radius curve will eventually move the tracks into a spiraled configuration, Figure 8. This, along with the superelevation, balances the centrifugal force going around the curve.

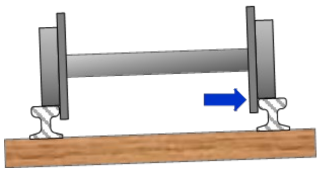


Figure 7

Instead of allowing them to move into a spiraled configuration, tracks are generally laid out using spirals in the first place.

Railroads are a specialized application; we'll only deal with highway applications.

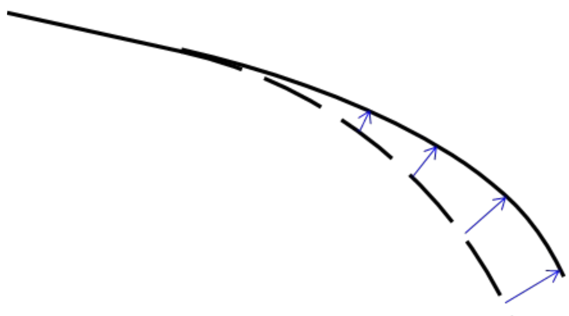


Figure 8

C Spiraled Curves

1. Transition Spirals

While it is possible to use an entrance spiral and an identical but reversed exit spiral, Figure 10, to transition between tangents that's generally not the case. Usually the spirals need only be long enough to introduce and remove superelevation. A circular curve arc is inserted between the spirals, Figure 9. This has the advantage of keeping the computations relatively simple.

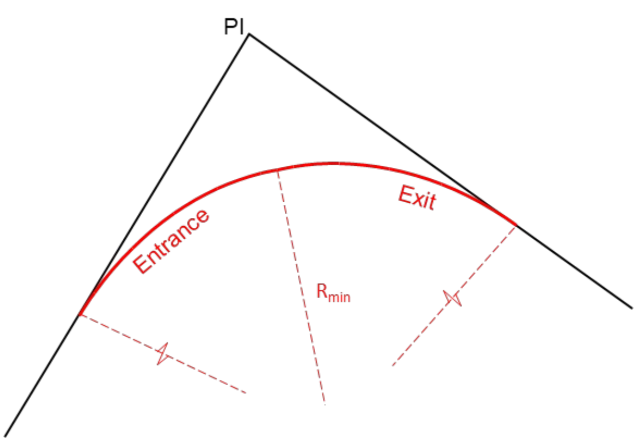


Figure 10

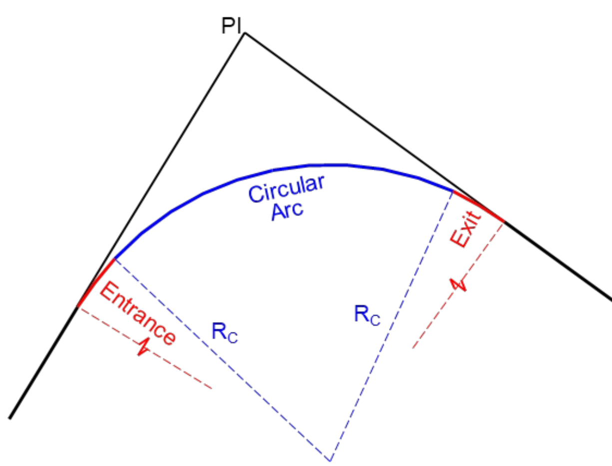


Figure 9

How short can the spirals be? Wisconsin's DOT *Facilities Design Manual* shows a transition distance of 270 feet at a 70 MPH design velocity for a 2040 ft minimum radius curve and 6% maximum superelevation[2]. That's a pretty sharp curve; typically a flatter curve needing a shorter transition distance would be used.

2. Cloverleaf

A traditional cloverleaf interchange changes travel direction by 270° or more depending on the road intersection geometry. Consider northbound traffic that will travel west, Figure 11.

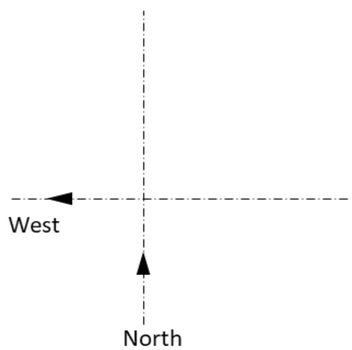


Figure 11

To keep traffic flowing, northbound traffic would turn to the right through a clockwise curvilinear transition. The traffic must slow from its normal speed, travel around a curved path, then speed up to match traffic on the westbound lane.

Using a tangent circular is disadvantageous because: (1) it is the least space efficient, and (2) it can be a very sharp curve requiring a lager superelvation and a substantial speed drop.

Another way is to use a multi-center compound curved; a common one is a five-center curve. This consists of a flat deceleration curve, followed by a sharper curve, followed by an even sharper curve for most of the directional transition, then a less-sharp curve, and finally a flatter acceleration curve. Although more space efficient than a single curve, if you've ever driven though one you can definitely feel when you're going from one curve to another.

A third option is to use a spiraled horizontal curve: an entrance spiral serves as a deceleration curve leading into a transitional fixed-radius circular arc, followed by an exit spiral which allows acceleration to match traffic speeds. This option is generally the most comfortable and space efficient.

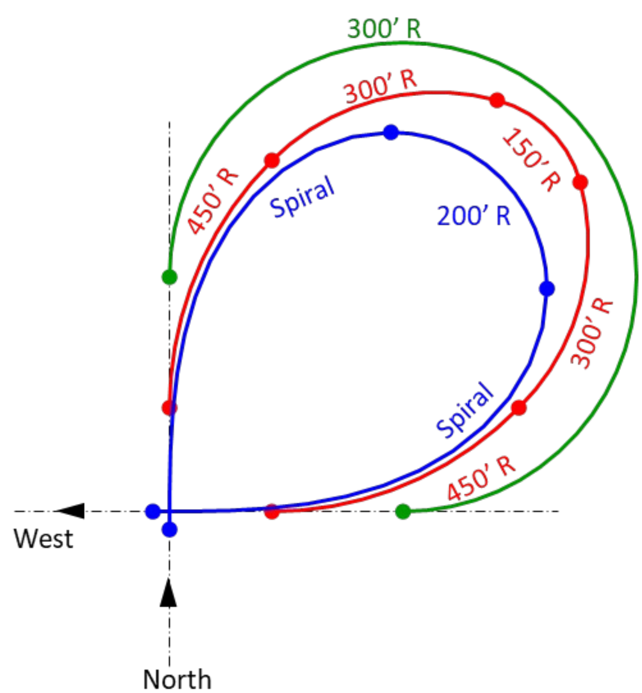


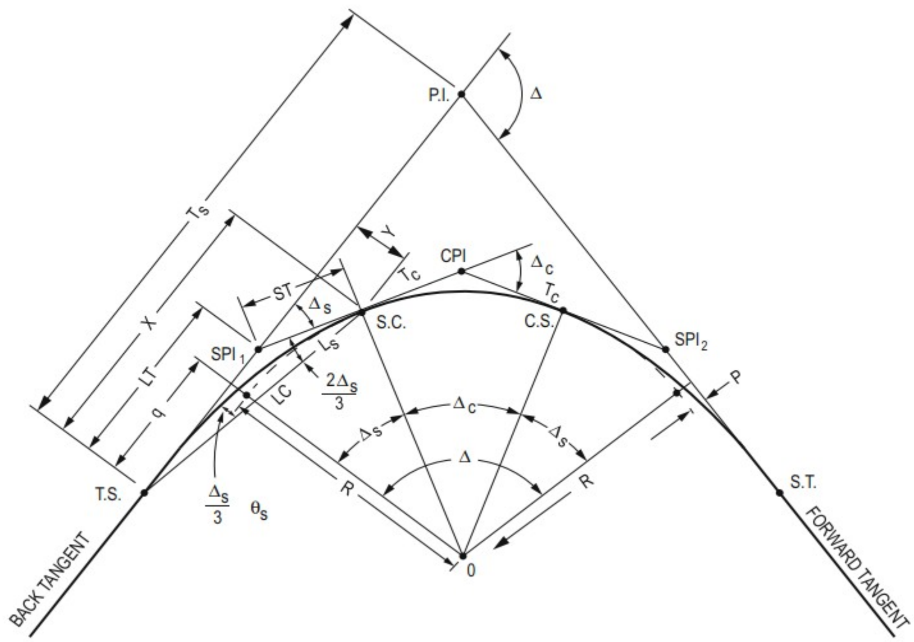
Figure 12

2 We have snow and ice here in Wisconsin so our superelevation rates are lower than southern states. It's not so much sliding **out** off a curve as sliding **in**.

All three configurations are shown in Figure 12.

II. Spirals: What All There Is

A. Classic Diagram



T.S. – Tangent to spiral

S.C. – Spiral to curve

C.S. – Curve to spiral

S.T. – Spiral to tangent

T_s – Spiral tangent

X – Distance along tangent from T.S. to point at right angle to S.C.

Y – Right angle distance from tangent to S.C.

LT – Long tangent (spiral)

ST – Short tangent (spiral)

L_s – Length of spiral (arc)

LC – Long chord

q – Distance along tangent to a point at right angle to ghost bc (marginally less than $L_s/2$)

P – Distance from tangent that the curve (ghost bc) has been offset

T_c – Circular curve tangent

CPI – Circular curve P.I.

SPI – Spiral curve P.I.

P.I. – Point of intersection of curve tangents

L_c – Length of circular curve

L – Length of curve system – T.S. to S.T.

Figure 13

The diagram in Figure 13 is typical of those found in text books and doesn't really instill confidence in understanding spirals. This particular rendition is taken from the NCEES *Principles and Practice of Surveying Reference Handbook Edition 1.2*[3]. There are a lot of components and parts but not all are needed to compute a basic spiral. Rather than try to decipher the entire diagram by staring at it until your eyes bleed, let's start with the basic parts and build it from there.

B. Inserting Spirals

Begin with a constant radius circular arc, R, that transitions through Δ degrees between two tangents, Figure 14

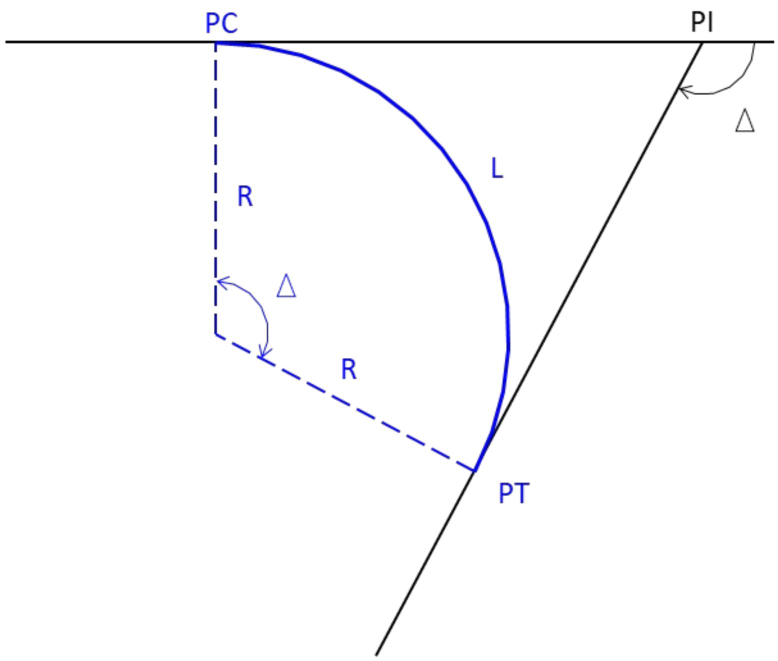


Figure 14

Maintaining the curve's fixed radius and adding entrance and exit spirals, Figure 15, causes two things to happen:

- The combined curves are shifted to the inside of the circular curve
- The circular curve length is shortened

3 Interestingly, the *Manual* doesn't contain any spiral equations.

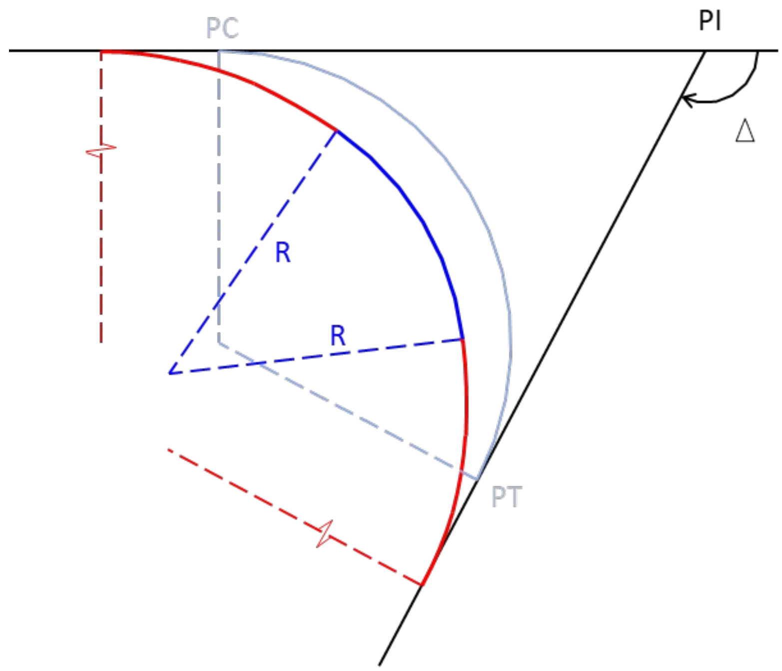


Figure 15

C. Nomenclature; Parts

1. Spiraled Curve

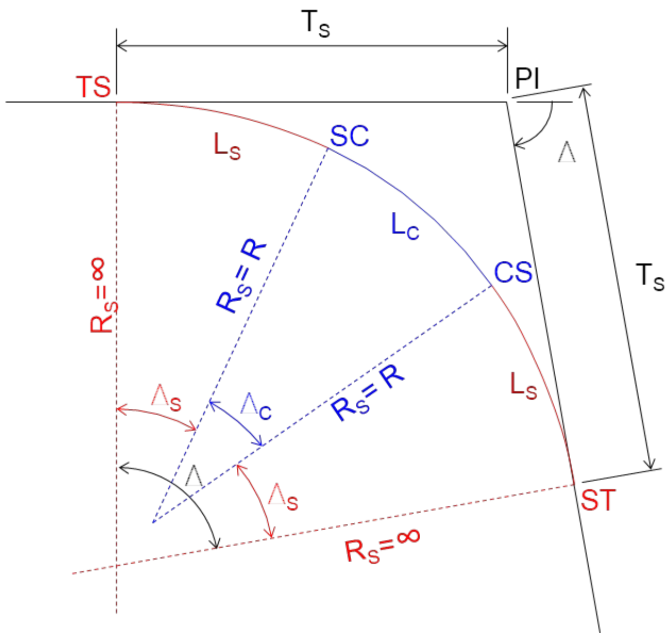


Figure 16

Generally, the defining geometric elements of a spiraled horizontal curve are:

Δ, the deflection angle between the tangents at the PI
Circular curve radius (R) or Degree of curvature (D)
Since these are highway curves, D is the arc definition
Spiral length, L_S

The entrance spiral begins with an infinite radius decreasing to R; the exit spiral radius increases from R to infinity.

The spiral's degree of curvature varies also. The rate of change is: $k = \frac{D_C}{L_S}$

Curve endpoints

TS	Tangent to Spiral	SC	Spiral to Curve
CS	Curve to Spiral	ST	Spiral to Tangent

Central angles

Δ_S Entrance and exit spirals, each
Δ_C Circular curve

$$\Delta = \Delta_C + 2\Delta_S$$

$$\rightarrow \Delta_C = \Delta - 2\Delta_S$$

$$\Delta_S = \frac{L_S D}{200}$$

Circular curve components, Figure 17, are computed normally using Δ_C in lieu of Δ.

Length:

$$L_C = \frac{100\Delta_C}{D}$$

Long Chord:

$$LC_C = 2R\sin\left(\Delta_C/2\right)$$

Tangent:

$$T_C = R\tan\left(\Delta_C/2\right)$$

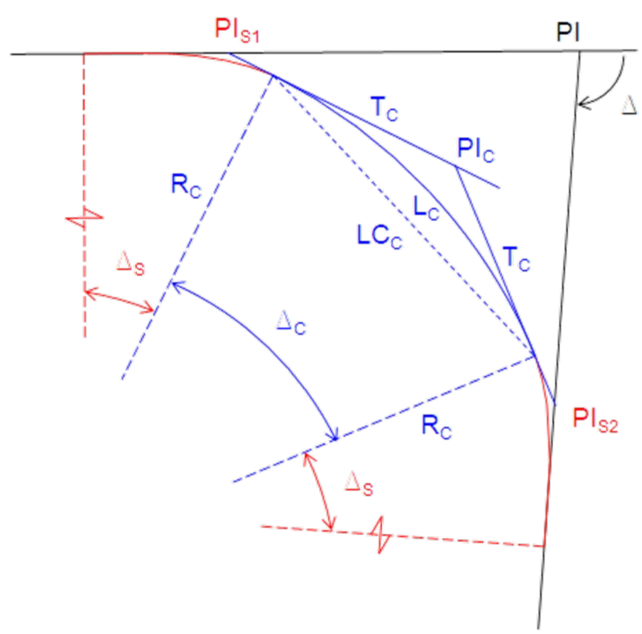


Figure 17

2. Stationing

Figure 18 looks a little complex, but that's only because there are so many pieces that make up T_S, the tangent distance from the TS to the PI (and PI to ST).

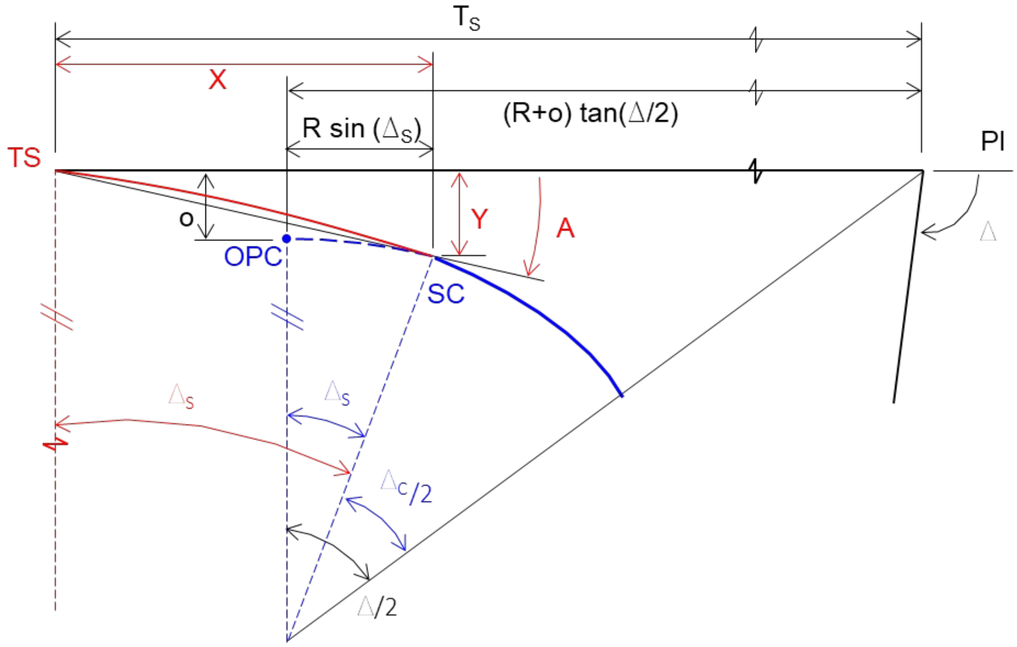


Figure 18

The OPC is the Offset PC. It's located by extending the circular arc back to where the radius line is perpendicular to the tangent line.

Angle A is the total deflection angle from the flat end of the spiral. Because the spiral is short and relatively flat, chord lengths be approximated with arc lengths.

$$A \approx \frac{\Delta_S}{3}$$

$$Y \approx L_S \sin\left(\frac{\Delta_S}{3}\right)$$

$$X \approx L_S - \left(\frac{Y^2}{2L_S}\right)$$

$$X_O = X - R_c \sin \Delta_S$$

$$o = Y - R_c\left(1 - \cos \Delta_S\right)$$

$$T_S = X - R_c \sin\left(\Delta_S\right) + \left(R_c + o\right) \tan\left(\Delta\!/\!_2\right)$$

Given all that, curve endpoint stations can be computed:

$$Sta_{TS} = Sta_{PI} - T_S$$
$$Sta_{SC} = Sta_{TS} + L_S$$
$$Sta_{CS} = Sta_{SC} + L_C$$
$$Sta_{ST} = Sta_{CS} + L_S \quad Back$$
$$Sta_{ST} = Sta_{PI} + T_S \quad Ahead$$

Actually, this is really the most involved part of spiral computations.

III. Spiral by Deflection Angles

The Deflection Angle Method for spiral computation and stake out is similar to that used for a regular horizontal curve. Because a spiral changes radius. the computations and stake out are done from the flat end of the spiral.

The total deflection angle at the flat end of the spiral is (Δ_S)/3, at the sharp end (2Δ_S)/3, Figure 19.

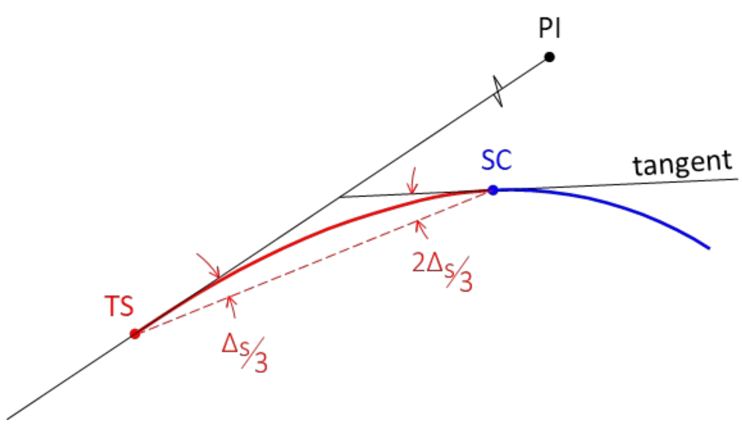


Figure 19

A. Approximate Method

This method uses arc distances as chord distances, Figure 20, which simplifies computations considerably.

The deflection angle is proportional to the square of the distance along the spiral.

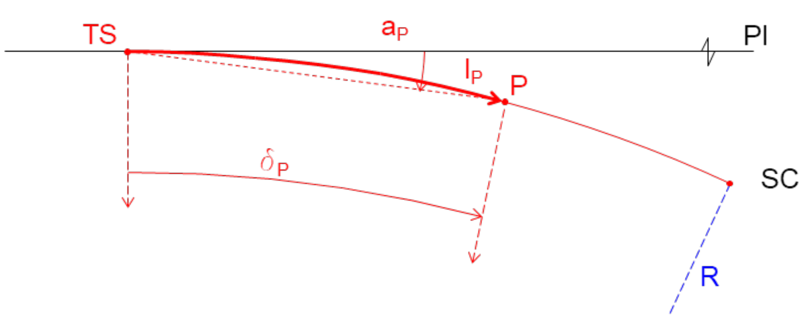


Figure 20

$$l_P = Sta_P - Sta_{TS}$$
$$a_P = \left[\frac{l_p}{L_S}\right]^2 \left[\frac{\Delta_S}{3}\right]$$

B. Tangent Offset Method

Curve point positions are located by the a distance along the tangent (x) and a right angle offset (y), Figure 21. This method also treats arc distances for chords.
The radial chord and deflection angle are computed from the x and y coordinates.

$$R_p = R_c \left[\frac{L_s}{l_p}\right]$$
$$y_P = \frac{l_p^3}{6 R_P L_S}$$
$$x_p = l_p - \frac{y_p^2}{2 l_p}$$
$$a_p = \tan^{-1}\left(\frac{y_p}{x_p}\right)$$
$$c_p = \sqrt{x_p^2 + y_p^2}$$

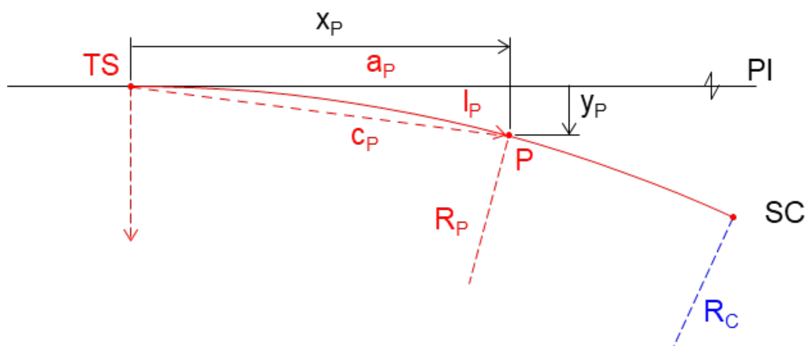


Figure 21

C. Power Series Method

The most way accurate to compute spiral points is using power series equations. These are a near exact method of determining the x and y distances of the tangent offset method, Figure 22.

$$\delta_P = \frac{l_P^2}{2 R_P L_S}$$
$$x_p = l_p \left[1 - \frac{\delta_p^2}{5(2!)} + \frac{\delta_p^4}{9(4!)} - \frac{\delta_p^6}{13(6!)} + \cdots\right]$$
$$y_P = l_p \left[\frac{\delta}{3} - \frac{\delta_p^3}{7(3!)} + \frac{\delta_p^5}{11(5!)} - \frac{\delta_p^7}{15(7!)} \cdots\right]$$

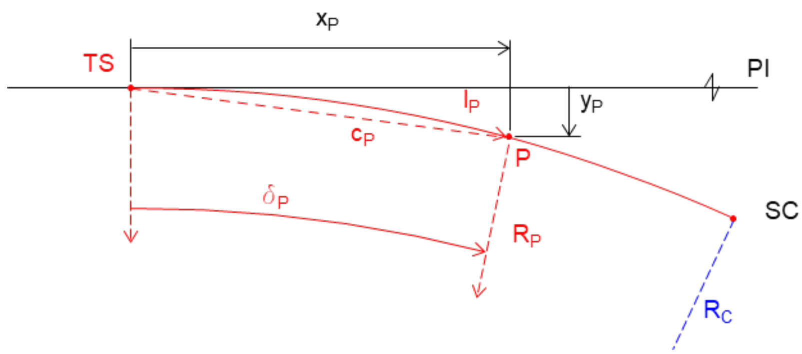


Figure 22

The x_p and y_p equations are infinite series but each successive term is progressively smaller. Using the first three or four terms is sufficient for both equations.

δ_P is the central angle in radians of the spiral from the flat end to curve point P.

The radial chord and deflection angle are computed same as for the Tangent Offset method.

$$c_p = \sqrt{x_p^2 + y_p^2}$$

$$a_p = \tan^{-1}\left(\frac{y_p}{x_p}\right)$$

Although a more accurate way to compute a spiral, unless using a long spiral, the Power Series is overkill; either the Approximate or Tangent Offset Methods will generally suffice.

D. Spiral interval

Transition spirals are short and the entrance/exit spirals are geometric mirror images of each other. Because of this, spirals are typically computed in equal intervals from the flat end instead of at full- or half-stations.

Either a five or ten equal intervals (aka, five- or ten-chord) is commonly used: a 200 ft spiral computed using five chords would be staked every 40 feet.

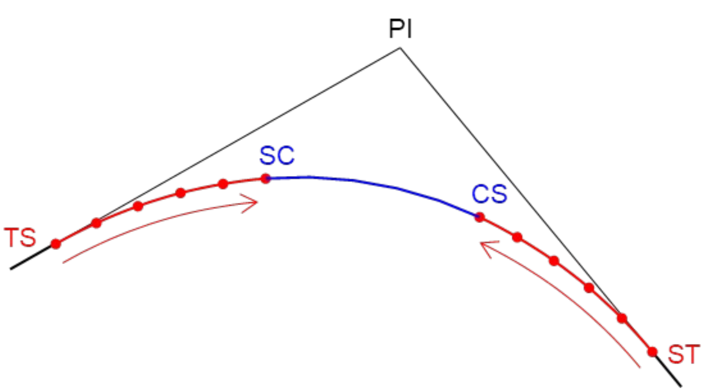


Figure 23

Although that results in "odd" stationing (eg, not +00 or +50) the trade-off is:

simpler computations.

one set of computations for both spirals

E. Circular Arc

The circular arc is computed using the regular horizontal curve Deflection Angle Method, Figure 24.

$$def\ \ rate = d\delta = {}^{D}\!/_{200}$$

$$l_p = Sta_P - Sta_{sc}$$

$$\delta_p = l_p \times d\delta$$

$$r_p = 2R_c \times \sin(\delta_p)$$

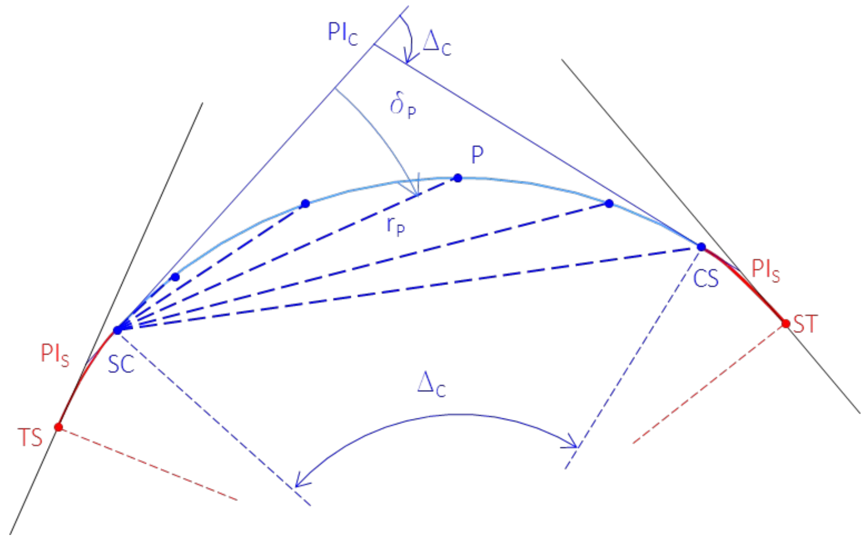


Figure 24

F. Traditional Stakeout

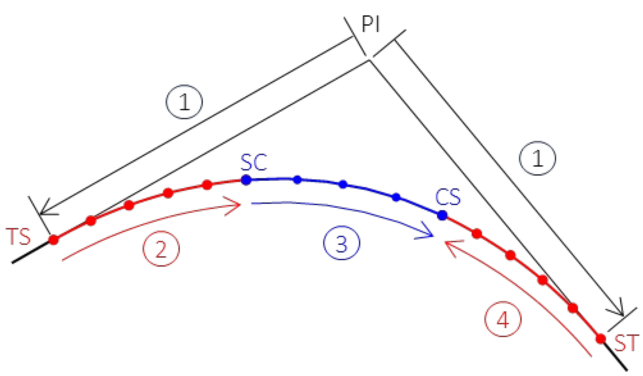


Figure 25

1. The TS and ST stations are set by measuring T_S along the tangent lines from the PI.

2. Stake entrance spiral from the TS

BS on PI
Turn to deflection angle and measure distance for the curve point
Continue to SC

3. Stake the circular curve from the SC to the CS

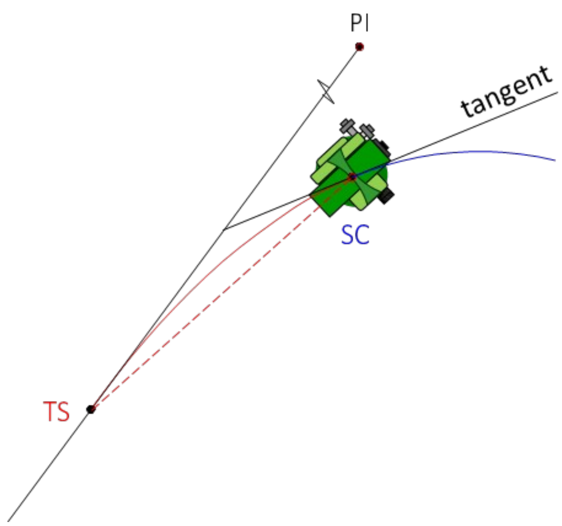


Figure 26

Set up on the SC.
BS the TS.

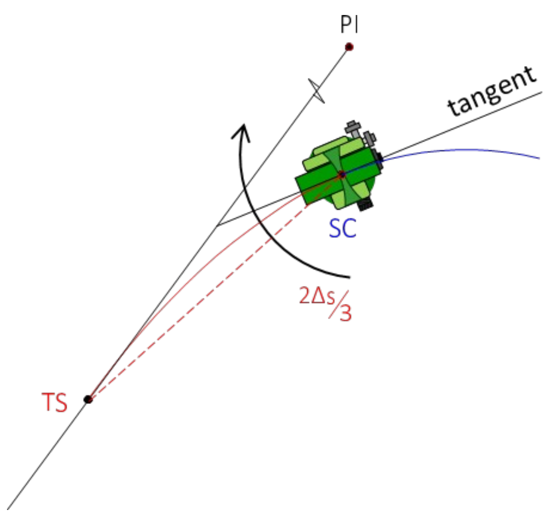


Figure 27

Rotate the instrument ($2\Delta_S$)/3. horizontally.
Are now oriented tangent to circular arc.

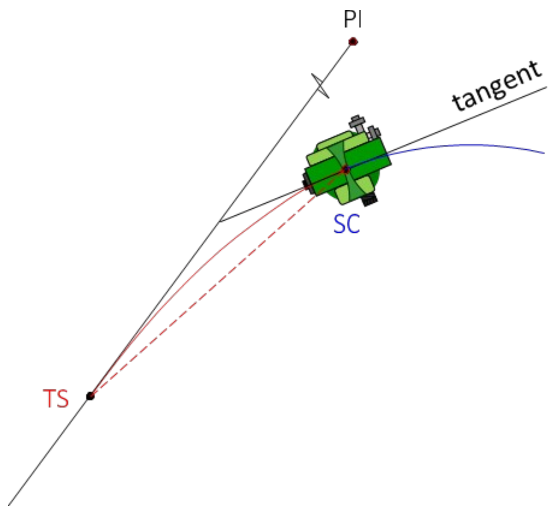


Figure 28

Rotate telescope 180° vertically.
Sighting tangent to circular arc.
Zero the horizontal angle.

Stake using deflection angle notes.

4. Stake exit spiral from the ST closing to the CS

   Use same procedure and notes as entrance spiral.
   Check against previously set CS stake.

IV. Examples

The example problems are part of the included Assignment.

V. Spreadsheet

jerrymahun.com

Home | Software | Excel Worksheets | Spiral Curve Computations

Primary Input	PI Station
	Total Δ
	Circular R
	Spiral Length
Computes	Spiral Components
	Arc Components
	Endpoints Stationing
	Five-Chord Notes
	Ten-Chord Notes
	Approximate Method
	Tangent Offset Method
	Power Series Method
Optional Input	Spiral distance
Computes	Deflection Angle
	Chord